Research results

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I'm studying stochastic analysis for jump-diffusion processes on Riemannian manifolds. The jump-diffusion processes are obtained by projecting a solution to the Marcus-type stochastic differential equation on the orthonormal frame bundle. Applebaum (1995) obtained the jump-diffusion process by using the Elles-Elworthy-Malliavin construction. We clarified the long-time behavior of jump-diffusion processes on Riemannian manifolds and obtained the Bismut-type integration by parts formula for the process. It is well known that one can verify that the existence of the law of the density function and that the smoothness of the density by using an integration by parts formula. Thus, it is expected that the existence and smoothness of the density function of the jump-diffusion process on manifolds can be proved. Moreover, by applying the Bismut-Elworthy-Li-type integration by parts formula for the solution to jump-type stochastic differential equations on the Euclidian space, which is obtained by Takeuchi (2010), we get the Bismut-Elworthy-Li-type formula for jump-diffusion processes on Riemannian manifolds. These results are published in Kai-Takeuchi (2021).

In the study of the long-time behavior of jump-diffusion processes on Riemannian manifolds, we showed that jump-diffusion process is the irreducible, transient, and conservative under the suitable assumptions on the Levy measure and the curvature of the manifold. We showed the irreducibility in terms of the functional analysis approach. Therefore, it is expected that we can prove the irreducibility of a Markov process whose generator is more general than that of the jump-diffusion process. Furthermore, by using the comparison theorem on the Hessian, we obtained an evaluation of the radial part of the jump-diffusion process on Riemannian manifolds. From this evaluation, we can prove that the jump-diffusion processes on Hadamard manifolds whose sectional curvature is pinched by two strictly negative constants are transient and conservative. The radial part of the Brownian motion is called the Bessel process, and there are so many works on this process. However, the radial part of the jump-diffusion process has not been well studied. Especially, few studies for the radial part of jump-diffusion processes exist from the viewpoint of stochastic differential equations, so the results of this study are the first step to understand how the curvature of the manifold affects the behavior of stochastic processes including jumps. The results of this study are currently scheduled for publication (#).

H. Kai and A. Takeuchi.: Gradient formula for jump processes on manifolds, Electron. J. Probab. **26** (2021), no. 101, 1 – 15.

H. Kai and A. Takeuchi.: Integration by parts formula on solutions to stochastic differential equations with jumps on Riemannian manifolds, J. Stoch. Anal. Vol 2 (2021)

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H. Kai.: Long time behavior of jump-diffusion processes on manifolds, Osaka J. Math. accepted on December 19, 2022.