Summary of Recent Research Results

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1 Local moves for classical knots and links

We considered several local moves for classical knots and links such as a crossing change, an H(2)-move, a coherent band surgery, and so on. From these moves we can define distances between knots or links. For example, the coherent band-Gordian distance between two links is the minimum number of coherent band surgeries needed to transform one into the other. This number is studied related to DNA topology, that is, smoothing a crossing may be the pathway for a knotted circular DNA to unknot itself.

We gave several criteria of links concerning these local moves in terms of determinants, special values of polynomial invariants, and so on, which allow us to decide the corresponding distances for several pairs of knots or links. In particular, we made a table of the H(2)-Gordian distances and coherent band-Gordian distances between knots and links with up to seven crossings.

Last year, we began to study the 4-move, which changes 4 half twists to 0 half twists or vice versa. There is an open problem proposed by Yasutaka Nakanishi in 1979, which asks if every knot is unknotted by a sequence of 4-moves. In a joint work with Hideo Takioka we studied the 4-move distances between knots.

2 Enumeration and classification of ribbon 2-knots

A ribbon 2-knot is a knotted 2-sphere in $\mathbb{R}^4$ that bounds a ribbon 3-disk, which is an immersed 3-disk with only ribbon singularities. The ribbon crossing number of a ribbon 2-knot is the minimal number of the ribbon singularities of any ribbon 3-disk bounding the knot. A ribbon 2-knot is constructed by adding r 1-handles to a trivial 2-link with $r + 1$ components for some r, which is called a ribbon 2-knot of r-fusion. A ribbon 2-knot is presented by a virtual arc diagram. with n classical crossings, then its ribbon crossing number is at most n.

We applied the twisted Alexander polynomial to classify ribbon 2-knots of 1-fusion and studied some properties of them. We enumerated ribbon 2-knots presented by virtual arc diagrams with up to 4 classical crossings, which we classified using the 2-fold branched covering space, the Alexander polynomial, the number of representations of the knot group to $\mathrm{SL}(2, \mathbb{F})$, $\mathbb{F}$ a finite field, and the twisted Alexander polynomial. Furthermore, we classified ribbon 2-knots with ribbon crossing number up to 4. These ribbon 2-knots are either of 1-fusion or the compositions of those of 1-fusion. On the other hand, we classified ribbon 2-knots of 1-fusion with small ribbon crossing number, and studied properties of the twisted Alexander polynomial and the 1-fusion presentation.

Last year in a joint work with Toshio Sumi we constructed examples of pairs of ribbon 2-knots with one fusion sharing isomorphic knot groups.