Future Research Plan
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My plan consists of two parts: continuing the current research [List 1,3,6] and exploring the new research interests. To find prospective topics, attending seminars regularly and communicating with experts in unfamiliar research areas will be valuable. Living in Japan would help those desires since Japan has so many conferences open to all mathematicians to attend and to communicate each other.

My current on-going research includes:

1. Line graph operator on directed graphs [List 1]:
   Characterize pre-periodic graphs together with dissipating waves and the fundamental domains;

Quasi-surfaces T_k: Note that T_1 is homeomorphic to the 2-disc with boundary, called Event Horizon, and $T_2 \cong S^2$ the 2-sphere with the equator as Event Horizon. For $k \geq 3$, we can prepare disjoint k 2-discs with boundary homeomorphic to S^1, and we can obtain T_k by gluing *k* boundaries one by one by using the corresponding homeomorphism from each boundary to S^1. Also notice that if we puncture at a point of Event Horizon of T_k, then the resulting topological space is homeomorphic to the book space having *k* pages.

2. Quasi-surfaces T_k and their topological properties [List 6]:
   How should we understand the fact that for $k \geq 3$, T_k is embeddable in $\mathbb{R}^3$ (orientable) but itself is non-orientable? Is this similar to the fact that the 2-dimensional hyperbolic (non-Euclidean) geometry model is embeddable in Euclidean geometry $\mathbb{R}^2$?

3. Quasi-surfaces T_k and varieties of embedding of graphs in T_k [List 3]:
   For a given graph G, determine a set M(G) of numbers *k* such that G is embeddable into T_k with a certain embedding method M and without excessive open 2-discs (hemispheres). Investigate the set M(G): Does M(G) contain a gap between minimum and maximum of M(G)? For each k in M(G), how many different embeddings are there? If we focus on 2-cell embedding for M, I believe that M(G) consists of at most two elements and their difference is at most 1. Thus, my conjecture is that no gap between min and max of M(G).

My new research interests include investigating how close the relationship between (A) algebraic graph theory using adjacent matrices and (T) topological graph theory each other. For example, the following properties: graph embedding(T), especially triangulations(T), graph minors(T), chromatic numbers(T), distance regularity(A), distance-polynomial(A), super-regularity(A), polynomial and co-polynomial scheme(A). Those algebras are developed based on adjacency matrices of graphs.