

# Research Plan

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I have three ongoing research projects.

The first one is to study injective homomorphisms between finite index subgroups of the mapping class groups of surfaces. In order to deepen our knowledge about such injective homomorphisms, I will investigate how injective homomorphisms from pure braid groups are restricted from the topological point of view. In this regarding, I found that the embedded image of each generator of a pure braid group in the mapping class group of a low genus surface is “reducible” and of specific form. I would like to generalize this fact.

The second one is to figure out the relation between invariants of flag complexes and injective homomorphisms of RAAGs. I constructed an injective continuous map between the flag complexes of the defining graphs of RAAGs  $A_1$  and  $A_2$  if  $A_1$  is embedded in  $A_2$ . I would like to study the induced homomorphism between the homotopy groups of those flag complexes.

(Joint work with Erika Kuno) The last one is to define the marking complex for a non-orientable surface and to find its application. The marking complex for an orientable surface is defined by Masur–Minsky, and this complex played an important role in their theory on the mapping class groups of surfaces. For example, the Cayley graph of the mapping class group of an orientable surface is quasi-isometric to the marking complex. For some technical (and sentimental) reasons, the study of the mapping class group of non-orientable surfaces is rather undeveloped. Therefore we planed to define the marking complex for a non-orientable surface. We have two candidates of the marking complex, so we would like to see they are successful or not. If successful, we can use the marking complex to study the mapping class group of a non-orientable surface from the geometrical point of view.