Research outline

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I have been working on geometric group theory. Geometric group theory is an area devoted to studying algebraic and geometric properties of groups endowed with the structure of a metric space, given by the so-called word metric. Especially, I have studied injective homomorphisms from right-angled Artin groups (RAAGs) into various groups and their applications by using topology and graph theory. Here, the RAAG of a finite (non-oriented) graph is the group such that the generators are the vertices, and the relators are the commutators of a pair of vertices that form an edge. As graphs are used to define RAAGs, various researchers have been studying relation between graph theory and algebraic and geometric properties of RAAGs.

(1) On injective homomorphisms from RAAGs

For every RAAG and knot in S^3, I decided whether the RAAG is embedded in the fundamental group of the knot complement. The knot complement can be decomposed into a union of small 3-manifolds that are easy to study by JSJ decomposition. I obtained the above result by using JSJ decomposition and reducing the problem on general knot complements to the problems on specific 3-manifolds. I also studied injective homomorphisms between RAAGs. I proved that we can construct a full injective graph homomorphism between the defining graphs from an injective group homomorphism from RAAG of the complement graph of a linear forest into the RAAG of a graph. This result is obtained by using normal form theory for (injective) homomorphisms between RAAGs and constructing explicitly a non-trivial element of the kernel of a group homomorphism between RAAGs. Also, the result led me to give necessary and sufficient conditions for embedding RAAGs of the complement graphs of line graphs into the mapping class group of orientable surfaces.

(2) On mapping class groups of surfaces

(Joint work with Erika Kuno) We gave necessary and sufficient conditions for embedding pure braid groups, finite index subgroups of the Artin's braid group, into the mapping class groups of orientable surfaces by using results in (1). It is known that the almost all orientation-double-coverings induce injective homomorphisms $(*)$ between the mapping class groups of surfaces. On the other hand, Koberda gave a method for embedding RAAGs into the mapping class groups of orientable surfaces. By combining these results, we gave a method for embedding RAAGs into the mapping class groups of non-orientable surfaces. Furthermore, we proved that the injective homomorphisms $(*)$ are quasi-isometric embeddings by using theory of semi-hyperbolic groups. We also gave an alternative proof of the Hamenstädt's theorem that states inclusion maps between surfaces induce quasi-isometric embeddings of the mapping class groups. Though the Hamenstädt's theorem is quite natural, I gave an alternative proof because her original paper has not been published over a decade.