Future plan

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Main achievements are construction of smooth maps whose codimensions are not positive of explicit nice classes and the manifolds, and studies on restrictions. In our studies, we have encountered various fields such as singularity theory of differentiable maps, algebraic topology, differential topology, geometry of low dimensional spaces, combinatoris etc.. We will study the following.

1. Generalizations of special generic maps. In our Preprint 15, we have introduced generalized special generic maps. We will study further. We expect these maps have nice properties on the structures as special generic maps and cover wider classes of manifolds. Special generic maps often restrict classes of manifolds of the domains. In addition, we respect local structures of so-called moment maps on some compact toric manifolds. Such manifolds admit no special geneirc maps in considerablel cases.

2. Geometric understanding on explicit higher dimensional manifolds via fold maps and more general smooth maps. For example, we are trying to understand from 5 to 8-dimensional manifolds via explicit fold maps or more general smooth maps. There are geometric results on such manifolds and we can also say that they are still unknown in many respects. Related Preprints are 3–5 etc.. Applying "1." before and some arguments expected to be closely related, we will continue trying to understand higher dimensions geometric and constructive. We are also interested in manifolds of dimensions at most 4. Among our Refereed papers, 6 shows that graph manifolds, forming a nice class of 3-dimensional closed and orientable manifolds, admit round fold maps into the plane. As a further study, we have found interesting relationship between the manifolds and the Reeb spaces, which are 2-dimensional polyhedra representing the manifolds compactly, in 8 of our Preprints. We will discover variants in higher dimensions.

3. Contributions to symmetric spaces. Symmetric spaces are smooth maps having nice symmetries. Such spaces and variants are important especially in differential geometry, Antipodal sets and variants have been shown to be characteristic finite sets and expected to be closely related to topological properties of the spaces. Nice classes such as the class of R-spaces, maximal antipodal sets are realized as the set of all singular points of some nice Morse functions. We expect our global theory of nice smooth maps to give strong tools. Our Refereed paper 7, constructing real algebraic functions explicitly, is expected to be very important.