Our achievements

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We are studying the following fundamental and important problem: understanding the topologies and the differentiable structures of manifolds. We are interested in using nice smooth maps into spaces of lower dimensions, especially, Morse functions, fold maps, and more general nice smooth maps. This gives well-known methods. Note that fold maps are smooth maps around singular points of which they are the produts maps of Morse functions and the identity maps on spaces of suitable dimensions. The following show some of our achievements. Some preprints are on the well-known preprint server https://arxiv.org/, submitted to refereed journals or under reviews with positive comments .

1. Some suitable classes of fold maps. In Refereed papers 1–3 and 8 and Preprint 1, we have defined round fold maps as fold maps such that the set of all singular points are embedded concentrically in the spaces of the targets and studied algebraic topological and differential topological properties of them and the manifolds.. Recently, we are studying speciaal generic maps: Morse functions with exactly two singular points on spheres and canonical projections are generalized as such maps. We have studied restrictions on the cohomology rings of the manifolds explicitly. Since the 1990s, Osamu Saeki, our host professor, etc. have studied restrictions on the differentiable structures of spheres or more general manifolds and the homology groups. Cohomology gives more precise information than homology groups by having structures of products. Preprints 6, 7, 16 etc. are on fundamental theorems. 11–14 are on explicit restrictions on the cohomology rings for explicit classes of higher dimensional closed and simply-connected manifolds, which are central objects in classical manifold theory, and here fundamental and new methods in differential topology work well.

2. Reeb spaces and various applications. The Reeb space of a smooth map is the space of all connected components of all preimages. In general, they are polyhedra whose dimensions are equal to the dimensions of the spaces of the targets and have some infomration of manifolds. In our Refereed papers 4, 5 and 7, a problem proposed by Sharko has been studied: for a given graph can we construct nice smooth functions whose Reeb graphs are the given graphs? Such functions on closed surfaces are well-known for example. We have studied cases the dimensions of the manifolds are general and the manifolds are not compact. Our preprints 2, 9 and 10 are also related.