

FUTURE RESEARCH

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1. INTRODUCTION

The problem of determining if a given Lie algebra admits a symplectic structure remains difficult in general and the picture seems far from complete. Only some classifications in low dimensions and some special cases in higher dimensions are known. (See [1] and references within)

We want to continue to develop the ideas introduced in [2] and [3]. We want to advance the understanding of symplectic Lie algebras and to obtain more classification results. In particular we have the following problems.

2. FUTURE PROBLEMS

2.1. Completing the classification of left-invariant symplectic structures on almost abelian Lie algebra. In [2] we studied a particular family of almost abelian Lie algebras whose adjoint homomorphism is diagonalizable, and obtained a classification of symplectic structures. A natural step will be to use the same ideas to study the general case. For this it will be enough to consider almost abelian Lie algebras whose adjoint homomorphism is given in Jordan normal form.

2.2. Classification of left-invariant symplectic structures in other other cases. In low dimensions only the classification of the 4 dimensional case is complete. For six dimensional solvable Lie algebras only a partial classification is known. We would like to apply our method to complete this classification. Also we would like to explore other interesting higher dimensional cases.

2.3. Geometry of the orbit space and left-invariant symplectic forms. Our method for studying left-invariant symplectic forms is based on the study of the moduli space of nondegenerate 2-forms. This is the orbit space of certain action and is a pseudo-Riemannian symmetric space. We want to investigate if symplectic forms correspond to distinguished orbits in this space.

REFERENCES

- [1] Baues, O. and Cortés, V. : *Symplectic Lie groups*. Astérisque 379 (2016).
- [2] Castellanos Moscoso, L. P.: *Left-invariant symplectic structures on diagonal almost abelian Lie groups*. Hiroshima Math. J. 52 (3) 357–378 (2022).
- [3] Castellanos Moscoso, L. P. and Tamaru, H.: *A classification of left-invariant symplectic structures on some Lie groups*. Beitr Algebra Geom.(2022).

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