RESEARCH RESULTS SO FAR

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1. Introduction

The problem of determining whether a given manifold admits a symplectic structure is a classical and hard problem. In the setting of Lie groups, it is natural to ask about the existence of left-invariant structures. A *symplectic Lie group* is a Lie group G endowed with a left-invariant symplectic form ω (that is, a nondegenerate closed 2-form). The study of symplectic Lie groups reduces to the study of *symplectic Lie algebras* $(\mathfrak{g}, \omega)$, that is Lie algebras $\mathfrak{g}$ endowed with nondegenerate closed 2-forms (or equivalently two-cocycles $\omega \in Z^2(\mathfrak{g})$). Still the problem of determining if a given Lie algebra admits a symplectic structre remains difficult in general and the picture seems far from complete. Only some classifications in low dimensions and some special cases in higher dimensions are known.

In [4] we can find a novel method to find nice (e.g., Einstein or Ricci soliton) left-invariant Riemannian metrics. The method is based on the moduli space of left-invariant Riemannian metrics on a Lie group G (the orbit of space of certain group action). In [5] and [6] the authors adapted the same ideas in the pseudo-Riemannian case succesfully. It was natural then, to try to use the same ideas for symplectic Lie groups.

2. Results

(1) Inspired by those previous studies in [3] we developed a similar approach for the study of symplectic Lie groups. We study the moduli space of left-invariant nondegenerate 2-forms on Lie algebras $\mathfrak{g}$.

(2) As a first application of these ideas also in [3] we studied two particular Lie algebras: the Lie algebra of the real hyperbolic space $\mathfrak{g}_{\mathbb{RH}^{2n}}$, and the direct sum of the 3-dimensional Heisenberg Lie algebra and the abelian Lie algebra $\mathfrak{h}^3 \oplus \mathbb{R}^{2n-3}$. We obtained a classification of symplectic structures on both of them.

(3) These two Lie algebras belong to a special family of Lie algebras: they are both almost abelian Lie algebras. In [2] we studied a particular family of almost abelian Lie algebras whose adjoint homomorphism is diagonalizable, and obtained a classification of symplectic structures.

(4) We obtained an improvement on two decomposition theorems of symplectic matrices: the symplectic QR decomposition and the SR decomposition.

References

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