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Research proposal

2D gravity and topological recursions. Moduli spaces of bordered Riemann surfaces are important objects to understand mathematical structures of 2D gravity and matrix models, and their volumes satisfy various relations. For example, the Weil-Petersson volumes satisfy not only the famous Mirzakhani's recursion, but also the Virasoro constraints and the CEO (Chekhov-Eynard-Orantin) topological recursion. The CEO topological recursion is applied to various models related to 2D gravity, and leads to, in 2017, the formulations of the ABO (Andersen-Borot-Orantin) topological recursion, which is a generalization of the Mirzakhani's recursion for the Weil-Petersson volumes, and the quantum Airy structures by Kontsevich-Soibelman and Andersen-Borot-Chekhov-Orantin, which give a generalization of the Virasoro constraints. By the generalizations, the models related to 2D gravity may be studied from the point of view of 2D complex geometrical approach by the CEO topological recursion, 2D geometric topological approach by the ABO topological recursion, and algebraic approach by the quantum Airy structures. Recently, by using these three approaches, I am studying, with Hiroyuki Fuji, the 2D $(2, p)$ minimal gravity with an odd integer p and its Masur-Veech type twist. Here, the volumes for the $(2, p)$ minimal gravity are considered as a generalization of the Weil-Petersson volumes for $p \to \infty$, and the Masur-Veech type twist provides statistics of lengths of multicurves on Riemann surfaces. In the future, I want to study other models related to 2D gravity by applying these approaches.

3D supersymmetric gauge theory and knot theory. We have constructed an abelian gauge theory in [20] of List of Publications, that we called "knot-gauge theory", whose K-theoretic vortex partition functions give the colored Jones polynomials of knots in S^3. For the construction, we utilized an exact formula of the A-twisted partition function (twisted index) of 3D $\mathcal{N} = 2$ gauge theory on $S^2 \times S^1$ obtained by Benini-Zaffaroni in 2015, and the factorization of the A-twisted partition function into the K-theoretic vortex partitions. About the knot-gauge theory, there are some problems to unveil in my research as follows. Firstly, the knot-gauge theory provides a relation "colored Jones polynomial = K-theoretic vortex partition", and the right hand side is expected to be obtained as a generating function of Euler characteristics for moduli spaces of vortices. So, it is interesting to construct the vortex moduli spaces and provide a geometric interpretation for the Jones polynomials. Secondly, the knot-gauge theory is labeled by tangle diagrams of knots and has R-matrix-based building blocks, and as a result, it is non-trivially transformed under the Reidemeister moves. It is important to clarify whether one can understand this transformation in terms of "3D dualities". Thirdly, it would be interesting to introduce the parameter t of homological grading, by Dunfield-Gukov-Rasmussen, that categorifies the colored Jones polynomial, to the R-matrix. It is not known, for general knots, how the parameter t is introduced, and this is a mathematically challenging problem. Our construction of the knot-gauge theory is based on the computation of the colored Jones polynomial by the R-matrix, and this is also related to giving an interpretation of the parameter t in the knot-gauge theory.

Non-perturbative topological strings and refined topological strings. The topological string theory is perturbatively well-defined, and it is important to understand its non-perturbative nature. For this research direction, in 2008, Eynard and Marino proposed a method to provide non-perturbative corrections to the perturbative free energies by the CEO topological recursion. On the other hand, it is also known that, for the refined (one-parameter deformed) topological string theory on local toric CY3s, the so called NS (Nekrasov-Shatashvili) limit provides a non-perturbative correction to the (unrefined) topological strings. It is interesting to clarify the relation between the proposal by Eynard-Marino and the non-perturbative corrections by the NS limit. Here, the refined topological string theory is also known to be related with the categorification of the colored Jones polynomial in knot theory, and it is interesting, in my research, to study the relations with the non-perturbative nature in the context of "geometric topology", "algebraic geometry", and "enumerative geometry". Furthermore, the brane partition function in the topological string theory gives a quantization of spectral curves in the target spaces, and the brane partition function in the refined topological string theory is considered to give a "double quantization" of spectral curves. I also want to understand the relation between the double quantization and the non-perturbative corrections.