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Research Accomplishments

I am a theoretical physicist, who is interested in mathematical physics, in particular, topological string theory, matrix models and supersymmetric gauge theories. By using various techniques in string theory and gauge theories, I have worked on exploring non-trivial relations between seemingly different mathematical (especially enumerative geometric) objects and developing their computational techniques. The following is an overview of some of my works. The reference numbers are the ones in "List of Publications".

CEO TR (topological recursion). The CEO TR by Chekhov-Eynard-Orantin can be considered, in general, for a 1D algebraic variety $\Sigma = \{ (x,y) \in \mathbb{C}^2 \,|\, A(x,y) = 0 \}$ which is called a spectral curve, and defines multilinear meromorphic differentials $\omega_h^{(g)}(z_1,\ldots,z_h)$, $h \geq 1$, $g \geq 0$ on Σ, from the Liouville one-form $\omega_1^{(0)}(z) = ydx$ and the Bergman kernel $\omega_2^{(0)}(z_1,z_2)$ that are (classically) defined for Σ, where z is a local coordinate on Σ: $x = x(z), y = y(z)$. The CEO TR has its origin in the loop equations of matrix models, and there are various interesting applications to the models related to the topological string theory and 2D gravity. I will summarize my works relevant to it:

- For a class of local toric Calabi-Yau 3-folds (CY3s), the differentials $\omega_h^{(g)}(z_1,\ldots,z_h)$ are identified with generating functions of the open Gromov-Witten invariants of genus g for certain Lagrangian submanifolds. Furthermore, around 2010, via geometric engineering, it was conjectured that they also give correlators for a surface operator in 4D $\mathcal{N}=2$ $SU(N)$ supersymmetric gauge theories. We explicitly checked and confirmed it for $SU(2)$ gauge theories [2,4].
- Around 2009, by Dijkgraaf and Fuji, it was proposed an embedding of the volume conjecture in the 3D $SL(2,\mathbb{C})$ Chern-Simons gauge theories into the topological string theory. Here, the moduli space of flat connections (the space of classical solutions) in a Chern-Simons gauge theory is described by an algebraic variety called a $SL(2,\mathbb{C})$-character variety, and for a knot complement in S^3 the character variety gives a 1D spectral curve. We conjectured that for the character varieties of knots, the CEO TR gives a (large color) asymptotic expansion of the colored Jones polynomials of knots, and checked it by non-trivial examples [3].
- A class of matrix models has free field realizations in 2D and is deeply related to 2D conformal field theories (CFTs). For instance, for such matrix models, the CEO TR is derived as the loop equations, and the loop equations are also shown to be equivalent to the Virasoro constraints by the generators of Virasoro algebra in 2D CFT. We showed that an infinite family of quantum (spectral) curves, associated with a spectral curves in a hermitian matrix model, can be explicitly constructed by the CEO TR and identified with Belavin-Polyakov-Zamolodchikov differential equations for an infinite family of Virasoro singular vectors in 2D CFT [8]. We also discussed a supersymmetric generalization of the quantum curves [9,13].

Research by exact partition functions. Triggered by the work by Pestun in 2007 about the exact computation of the partition functions and correlators in $\mathcal{N}=2$ supersymmetric gauge theories on S^4, the supersymmetric localization technique used there has been applied to obtain exact results in supersymmetric quantum field theories in various dimensions and background geometries. I will summarize my works achieved using such exact results:

- The topological string theory is defined by topological twists of 2D $\mathcal{N}=(2,2)$ non-linear sigma model (NLSM) that is realized at the IR fixed point of 2D $\mathcal{N}=(2,2)$ gauged linear sigma model (GLSM), and so the GLSM is related, as a "UV completion" of the NLSM, to the topological string theory. In 2012, Jockers-Kumar-Lapan-Morrison-Romo conjectured that, when a GLSM describes a CY manifold, the GLSM partition function on S^2 gives the exact Kähler potential on quantum Kähler moduli space of the CY manifold. We applied their method to CY4s and conjectured an exact formula of the Kähler potential on the quantum Kähler moduli space of CY4s [5].
- By the A-twisted GLSM partition functions, we achieved the followings: − Computation of the B-model Yukawa couplings on local toric CYs (a proposal of systematic introduction of the twisted masses to remedy the subtlety by the non-compactness of local toric CYs) [15]; − Computation of the Givental I-functions for some of the determinantal CYs via "factorization" of the partition function of the A-twisted GLSM [16]; − Construction of the off-shell Bethe wavefunctions in $\mathfrak{su}(N)$ XXX (XXZ) spin-chain, via the 2D (3D) Bethe/Gauge correspondence, as orbifold-type codimension-2 defects (generalization of earlier works in the case of $\mathfrak{su}(2)$) [17].
- Using the A-twisted partition functions (twisted indices) of 3D $\mathcal{N}=2$ gauge theories on $S^2 \times S^1$, we constructed and proposed an abelian gauge theory, that we called "knot-gauge theory", whose K-theoretic vortex partition functions give the colored Jones polynomials of knots in S^3 [20].