

(2) Study proposal

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(i) Aim of the study

The aim of the study is to understand algebro-geometric aspects of algebraic complex $K3$ surfaces, which we simply call a $K3$ surface. The study of geometry of $K3$ surfaces is deeply related to classical areas such as singularity theory and algebraic curve theory, and mathematical physics. It is necessary to study the period mappings and the Picard lattices for the understanding of algebro-geometric properties of $K3$ surfaces, particularly related to Picard lattices, it is unavoidable to understand the behaviour of curves on the surfaces. Related to analytic geometry and mathematical physics, it is a long-lived problem to characterize $K3$ surfaces from the view point of the moduli spaces of maps from a $K3$ surface to a Lie algebra. We consider the following problems:

Problems

1. The relation between algebro-topological properties of singularities and $K3$ surfaces.
2. The moduli space of maps from a $K3$ surface to a Lie algebra.
3. Weierstrass semi-groups of pointed curves in a $K3$ surface.

(ii) Study methods

Problem 1 Based on the numerical relation between the invariants of the Seifert form of quasi-homogeneous isolated singularities in $\mathbb{C}^3$ and the hyperbolic lattices that can polarize $K3$ surfaces, we would like to know some geometric relation between the singularities and the surfaces. Moreover, we would like to know the duality between the “mirror duality between the families of $K3$ surfaces” and “(some) duality between the Seifert forms of singularities”.

Problem 2 This is a joint work with Professor Jiryo Komeda in Kanagawa Institute of Technology. We study the following questions:

- Which Weierstrass semigroups can we find for pointed curves on a $K3$ surface which is obtained by various rational elliptic surfaces ?
- Can we find other criterion for a given numerical semigroup to be Weierstrass ?
- Can we give a characterisation of numerical semigroups of type 2 ?

Problem 3 The applicant showed that if two weight systems with four weights are coupling dual, then, associated families of $K3$ surfaces can be polytope-dual, and for some coupling pairs, the families moreover become lattice-dual.

We would like to know whether there exists some duality between lattices of families of $K3$ surfaces that are not lattice-dual but polytope-dual.

(iii) Aspects

The studies of singularities and of Lie algebras are to understand local properties whilst the study of $K3$ surfaces is global. But there should be deep relations between these different-looking two objects. By using the lattice theory and the algebraic geometry for subvarieties, the applicant expects that the above proposals help revealing the secret of those relations.