(2) Abstract of results Makiko Mase

Part I. The Seifert form and polarized $K3$ surfaces

In this study, we study relations between the Seifert form of quasi-homogeneous isolated singularities in $\mathbb{C}^3$ and hyperbolic lattices that can polarize projective $K3$ surface. In the preprint paper, we compute the real Seifert form of such singularities by using Saeki's method, and compare the invariants of the form and the hyperbolic lattices. Finally, we find a numerical relation between these invariants.

In the second part of the paper is devoted to characterize lattices that can polarize weighted $K3$ surfaces. We give a big table of lattices with some important invariants.

Part II. On Weierstrass semigroup of a pointed curve on a $K3$ surface (joint work with Professor Jiryo Komeda)

Recall that we have considered the follwing problem:

PROBLEM: For a given numerical semigroup H, i.e., a subset of $\mathbb{N}_0 := \{0\} \cap \mathbb{N}$ such that the complement $\mathbb{N}_0 \backslash H$ is finite, can one construct a pointed curve (C, P) lying on a $K3$ surface such that the Weierstrass semigroup $H(P)$ coincides with H ?

and we have shown the following results:

(1) Take the Fermat curve of degree $4n$ in $\mathbb{P}(1,1,4)$: $F_n : x^{4n} + y^{4n} + z^n = 0$ with a point $P_n = (1 : \zeta : 0)$, where $\zeta^{4n} = -1$ on it. By studying intersections of F_n with the branch locus, one can show that the pre-image $\widetilde{F_n}$ of F_n is lying on the $K3$ surface S. Moreover, the Weierstrass semigroup of the pre-image $\widetilde{P_n}$ of the point P_n is given by

$$H(\widetilde{P_n}) = \langle 2n,\, 8n-2,\, 12n-1 \rangle.$$

(2) Take the following curve of degree $4n$ in $\mathbb{P}(1,1,4)$: $F_n : x^{4n-4}z + y^{4n} + z^n = 0$ with a point $P = (1 : 0 : 0)$ on it. By studying intersections of F_n with the branch locus, one can show that the pre-image $\widetilde{F_n}$ of F_n is in the $K3$ surface S. Moreover, the Weierstrass semigroup of the pre-image $\widetilde{P_n}$ of the point P_n is given by

$$H(\widetilde{P}) = \left\langle \begin{array}{l} 8n-8,\, 8n-6,\, 8n-4,\, 8n-2,\, 8n, \\ 16n-5,\, 16n-3,\, 16n-1 \end{array} \right\rangle.$$

In our recent study, we are studying the NON-existence of algebraic curves on a $K3$ surface that admit given numerical semigroup.