Research Plan

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Constructing exact solutions of the Einstein-Dirac-Maxwell system using only one kind of spinor field

Background and Objectives : It is known that a large and strong cosmic magnetic field exists in the universe, and it is believed that there is some kind of current that generates the cosmic magnetic field. Such a cosmological model is considered to be a hybrid system of a cosmic magnetic field, a matter that generate the magnetic field and a gravitational field. The applicant has succeeded in constructing an exact solution by employing a two-component dust fluid or two kinds of spinor fields as the matter that generate the current. However, these are toy models because they are specific to the configuration of matter. The purpose of this study is to construct an exact solution for a hybrid system of a kind of spinor, a magnetic field and a gravitational field, aiming at a realistic model.

Significance : Many exact solutions for systems of a electromagnetic field without a source and gravitational field are known. However, few exact solutions with an electric source are known that are close to the real universe. Also, many exact solutions of the system of spinor and gravity are known, but only a few exact solutions of the system of spinor, electromagnetic field, and gravity field are known. Because a systems of electric currents, magnetic fields, and gravitational fields tend to be complex and it is difficult to construct solutions. The geometry of contact manifolds is compatible with the system of a current and a magnetic field. If we succeed in providing a realistic model by making effective use of this structure, we will be able to provide a more advanced understanding of the structure of the universe and at the same time show that the spin geometry on contact manifolds is a useful tool in general relativity.

Method : The electromagnetic field generated by one kind of spinor is time-varying and cannot generate a pure magnetic field, so the usual contact magnetic field is not compatible with this system. The applicant has found that a consistent system can be obtained by considering a non-static stationary spacetime with two 1-forms that have similar properties to a contact form and using them to define the electromagnetic field. However, this system is complex and it is not easy to obtain an exact solution. However, numerical calculations have led to the expectation that a solution may exist. The applicant would like to continue the research in the direction of constructing a solution based on geometrical considerations, such as refining the geometric structure mentioned above.

Year's plan : From January to March, the applicant will investigate as many candidate geometrical structures as possible that realize the exact solution described above, investigate their concrete examples, and find an explicit indication of the exact solution. Next, from April to June, the geometric properties are abstracted and investigate in as general a setting as possible. Finally, from July to September, the applicant will summarize the results and write a paper.

Explicit indication of Sasakian quasi Killing spinor

Background and Objectives: A Killing spinor is an important subject in spin geometry and mathematical physics, and it have been studied extensively. Killing spinors on spheres and hyperbolic spaces have been given explicit indications. On the other hand, it seems that no explicit indication has yet been given for the Sasakian quasi Killing spinor, which is a generalization of the Killing spinor on Sasakian manifolds. The purpose of this study is to obtain an explicit representation of the Sasakian quasi Killing spinor.

Significance : The applicant has studied the application of the Sasakian quasi Killing spinor in general relativity. It is expected to be useful in mathematical physics in the future. The explicit indication of the spinor is very helpful for those who want to understand its properties, and will be meaningful for all researchers who study the Sasakian quasi Killing spinor in the future.

Method : The partial differential equation that determines the Sasakian quasi Killing spinor reduces an ordinary differential equation under geodesic coordinates, so there is a possibility of quadrature. In addition, since the applicant have already found a simple representation of a particular Sasakian quasi Killing spinor in three dimensions, the applicant may be able to deduce a general representation based on it.

Year's plan : From August to October, the applicant will estimate the indications in general situations based on the special solutions already obtained and quadrature of the equations. From November to December, the results are summarized in a paper.