Research Results

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A contact manifold is a manifold of odd dimension equipped with a 1-form called a contact form. Sasakian manifolds, a kind of contact manifolds, have appeared in various fields of mathematical physics. The most famous example is probably AdS/CFT. Another famous example is the study of the properties of the magnetic field defined by considering the contact form as a gauge field. This magnetic field is called the contact magnetic field. While there are some applications of contact geometry to general relativity, there are few studies that focus on contact magnetic fields. By making effective use of the contact magnetic field, the applicant constructed an exact solution of a hybrid system of electromagnetic field, matter field interacting with electromagnetic field, and Einstein gravity field, and studied its properties. Since the hybrid system of electromagnetic field, matter field and gravitational field is generally very complicated and it is difficult to construct an exact solution, it is significant to provide a cosmological model that describes such a system exactly. The applicant's research theme is the application of contact geometry to general relativity. In the following, the two theme, (1) the study of static Sasakian spacetime and (2) the study of Gödel-type spacetime, are discussed.

(1)**A Study of Static Sasakian Spacetime**：The contact magnetic field is an electromagnetic field in which the contact form of the Sasakian manifold is regarded as a gauge field. When an electric current is distributed along the Reeb vector field, which is a metric dual of the contact form, the magnetic field generated by the current according to Maxwell's equation is found to be the contact magnetic field. Furthermore, if the current is generated by a charged dust fluid flowing along the Reeb vector field, the Lorentz force on the dust fluid from the contact magnetic field vanishes and the Reeb vector field is geodesic tangent, which constitutes an exact solution to the hybrid system of magnetic field and matter. A static Sasakian spacetime given by the direct product of a 3-dimensional Sasakian manifold and time $\mathbb{R}$. The applicant was able to construct an exact solution of the Einstein system consisting of a contact magnetic field, a charged dust fluid, as described above, and a gravitational field. This study has already been published[1][2]. On the other hand, although this study assumes that the electric current is generated by a charged dust fluid, it is expected that a similar situation can be described by a more fundamental field because the fluid is a macroscopic composite. Then, the applicant was able to construct an exact solution of the Einstein-Dirac-Maxwell system using a spinor field that interacts with the gauge field instead of a charged dust fluid. The spinor field is a special spinor field on the Sasaki manifold, called the Sasakian quasi Killing spinor, which is also new. These results will be published in the future.

(2)**A Study of Gödel-type spacetime**：The Gödel universe is an exact solution to the Einstein equations with a rotating dust fluid as matter, and has the property that there exists a closed causal curve. The Gödel-type spacetime is a spacetime admit a unit timelike Killing vector field whose orthogonal distribution is non-integrable, and is a generalization of the Gm̈odel universe. Lorentz Sasakian manifolds are Lorentian analogues of Sasakian manifolds, and their contact form is timelike. Therefore, the direct product of a 3-dimensional Lorentz Sasakian manifold and a 1-dimensional space $\mathbb{R}$ is a Gödel-type spacetime, which is a family of spacetimes containing a Gödel universe. In this spacetime, we consider a magnetic field whose gauge field is the contact form and a complex scalar field coupled with it. When this complex scalar field has a nontrivial vacuum expectation value as in the Higgs mechanism, it is found that the generated current is coordinated along the Reeb vector field and constitutes an exact solution of the Einstein-Maxwell-Scalar system, which is a hybrid system of the contact magnetic field, the scalar fields and the gravity field. In this case, the spacetime is a Gödel universe. Therefore, the Gödel universe can be understood as a vacuum in the E-M-S system. This result has already been published[3]. Furthermore, the applicant consider the E-M-S system with the weakened assumption that the gauge field is a specific timelike 1-form and the complex scalar field is a constant field in 4-dimensional distinguishing stationary spacetime, where the specific timelike 1-form is the 1-form which characterize the stationary structure. Under this settings the applicant could prove that ,essentially, the only solution is the one described above. Therefore, the Gödel universe is realized as a nontrivial vacuum in the E-M-S system in the wide class of 4-dimensional stationary spacetime. This result will be published in the future.

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[1]H.Ishihara,S.Matsuno. PTEP 2022.2 (2022): 023E01.

[2]arXiv:2012.02432 (2020).

[3]H.Ishihara,S.Matsuno. PTEP 2022.1 (2022): 013E02.