

## Research Plan --- Yasuo Matsushita

For 2023 Academic Year

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My basic and main research is again concerning the geometry on the indefinite metric spaces.

Results obtained in the 2022 Academic year:

1. It is shown that a Kodaira-Thurston 4-manifold admits a neutral Kaehler metric.
2. It is shown that neutral Kaehler metrics on compact complex tori and primary Kodaira surfaces are 4-dimensional Walker metric of neutral signature.

I will be trying to write drafts for submitting papers including these recent results, in 2023 academic year.

In the preset academic year, certain definitive results are obtained. For example, one of them is follows: The Kodaira-Thurston 4-manifold admits an Einstein-Kaehler metric of *neutral signature*. This fact is important, since they are the first example of symplectic manifolds that do not admit any positive definite Kaehler metrics, shown by Thurston.

New Issue for the New Academic Year:

There are certain possibilities of finding new 4-manifolds, similar to the Kodaira-Thurston 4-manifolds, if we focus our attentions to the *metrics of neutral signature*, rather than the positive definite ones.

It is known that connected sums of arbitrary odd numbers of complex projective surfaces and also arbitrary odd numbers of opposite complex projective surfaces admit a neutral metric of signature  $(+ + - -)$  (YM 1991). We try to construct such neutral metrics *explicitly* on such 4-manifolds of the connected sums.

My Basic Research Plan at present:

The counterexamples of the Goldberg Conjecture for pseudo-Rimannian manifolds in dimensions 6 and 8 have been reported from 2007 until 2017. For manifolds of the lowest dimensional case, dimension 4, the situations concerning the Goldberg Conjecture it are is quite different from the manifolds of higher dimensional cases.

Therefore, our research target will be focused on the 4-dimensional manifolds, if the Goldberg Conjecture is true or not.

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