Research Plan

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1. On weakly reflective submanifolds in symmetric spaces

Let G/K be a compact symmetric space and N a submanifold of G/K. In my research it was shown that if N is weakly reflective then the inverse image $\tilde{N}$ under the parallel transport map $\Phi_K : V_{\mathfrak{g}} \to G/K$ is also weakly reflective. Although $\tilde{N}$ is infinite dimensional, many techniques in the finite dimensional Euclidean case are still valid due to linearity of the Hilbert space $V_{\mathfrak{g}}$. In particular, recently it was shown that weakly reflective submanifolds generalize helicoids in 3-dimensional Euclidean spaces. Applying those techniques to the weakly reflective PF submanifold $\tilde{N}$ we study the structure, homogeneity and classification of weakly reflective submanifolds in G/K. To do this we first give a characterization of so-obtained weakly reflective submanifolds $\tilde{N}$ and show a necessary and sufficient condition for $\tilde{N}$ to be weakly reflective. This research is planed to be conducted in cooperation with Professor Takashi Sakai (Tokyo Metropolitan University).

2. Unique existence of minimal orbits in Hermann actions ([7] in preparation)

For a given isometric action of a Lie group on a Riemannian manifold it is a fundamental to determine its minimal orbits. It is known that for the isotropy representation of a compact symmetric space G/K there exists a unique minimal orbit in each strata of the stratification of the orbit types (Hirohashi-Song-Takagi-Tasaki 2000). A similar property also holds for the isotropy action of G/K and more generally for commutative Hermann actions (Ikawa 2011). The purpose of this research is to extend this result to the case of Hermann actions which are not commutative. First I describe the orbits spaces of non-commutative Hermann actions in terms of root systems. If the theorem holds then I will prove it. If it does not hold then I will show a counterexample. This problem is closely related to the problem in the case of affine Kac-Moody symmetric spaces mentioned below and considered to be an important problem.

3. Isotropy representations of affine Kac-Moody symmetric spaces

Affine Kac-Moody symmetric spaces are infinite dimensional symmetric spaces proposed by C.-L. Terng and established by E. Heintze, B. Popescu and W. Freyn based on Kac-Moody theory. Many similar properties between those spaces and finite dimensional Riemannian symmetric spaces are known. In particular their isotropy representations are described by path group actions induced by Hermann actions or σ-actions studied in [5] and [6]. In this research I study the unique existence for minimal orbits in the isotropy representations of affine Kac-Moody symmetric spaces. In the finite dimensional case it is known that there exists a unique minimal orbit in each strata of the stratification of orbit types. I conjecture that the similar property also holds for affine Kac-Moody symmetric spaces. I aim to investigate and prove this conjecture. Moreover I study the symmetries of those minimal orbits, compare them with those in the finite dimensional case and clarify the differences and similarities.

4. Reformulation of soliton theories by affine Kac-Moody groups

According to the research by M. Kashiwara, M. Jimbo, E. Date and T. Miwa, the symmetries of soliton equations are described in terms of affine Kac-Moody algebras. On the other hand, C.-L. Terng and K. Uhlenbeck showed that transformations on solution spaces of some integrable equations can be described by loop group actions. These two researches have been conducted independently and the relation of those is not clear. An *affine Kac-Moody group* is the group corresponding to an affine Kac-Moody algebra and recently its realization is established in the framework of affine Kac-Moody symmetric spaces. An affine Kac-Moody group is realized as a T^2-bundle over a twisted loop group of smooth loops, which has a structure of tame Fréchet manifold (Hamilton 1982) and seems to be useful to study the connection between those two soliton theories. I aim to integrate the above two methods for soliton theories by affine Kac-Moody groups.