Results of research

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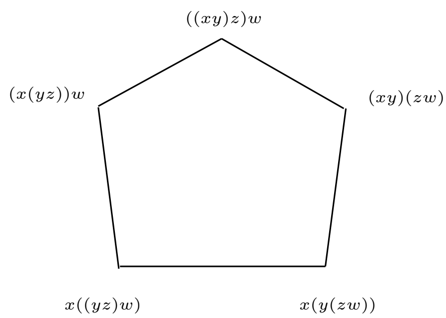
My research consists of the following two independent subjects.

1. non-simply connected rational homotopy theory(papers no. 5 and 6) The rational homotopy theory by Quillen and Sullivan states one-to-one correspondence between simply connected spaces and commutative differential graded algebras (in short, dg-algebras). By this correspondence, one can solve geometric problems after replacing it with algebraic problems, so the theory has many applications. The strongest assumption on this theory is the simple connectivity, which says the fundamental group is trivial. (More precisely, the theory is applicable to bit more general spaces but we omit details for simplicity.) I adopted a tensor dg-category as an algebraic model instead of a commutative dg-algebra to generalize the rational homotopy theory to non-simply connected spaces. I constructed this generalization refering to Toën's schematic homotopy type, and the tensor dg-categories which I used as models of spaces are equivalent to schematic homotopy types. My theory has two features. The first one is that the tensor dg-categories appear naturally in non-abelian Hodge theory due to Simpson, so it may fit the theory well. The second one is that homotopy invariants such as the Postnikov invariants and the action of fundamental group on homotopy groups are explicitly reconstructed from the algebraic model in view of Simpson's extensions of objects of a dg-category. In the original rational homotopy theory, it is important to give an algebraic explicit procedure to construct algebraic models mirroring a geometric procedure to construct a space (such as loop space) from a given space. Among important examples of these procedures, I generalized the model for loop spaces and cell attachments to non-simply connected spaces which satisfy certain conditions. While my generalization must impose some condition on the fundamental group comparing the generalization of Goméz-Tato–Halperin–Tanré, it has computational merit.

2. operads and spaces of knots (papers no. 1-4) Products naturally appearing in topology often satisfy the associativity law $(xy)z = x(yz)$ only up to homotopy. Many of such products intrinsically have certain system of higher homotopies. An operad is a framework which deals with such systems of homotopies associated to algebraic laws. Since 2000's, deep relation between operads and embedding spaces has been discovered. Lambrechts-Turchin-Volić gave combinatorial presentation of homology of the space of long knots in $\mathbb{R}^n$ with $n \geq 4$ using a graph complex, which gave an affirmative solution of a conjecture of Vassiliev in the rational, codimension ≥ 3 case.



I gave a simplified proof of the main theorem of Lambrechts-Turchin-Volić and slightly generalized it in the paper no.4. In the proof of the theorem of Lambrechts et al, formality of little disks operads is used essentially. In the paper no.2, I proved the framed little disks operads of dimension odd and ≥ 5 is not formal. In the paper no.3 (joint work with Keiichi Sakai), we proved affirmatively a conjecture due to Mostovoy, which states the classifying space of the space of long knots in $\mathbb{R}^3$ is homotopy equivalent to the space of short ropes. In the paper no.1, I constructed a spectral sequence for cohomology of the space of knots in a manifold and computed some examples.