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Research plan

We have studied the structure of the categories of modules of the triplet W-algebras and the $N = 1$-supertriplet W-algebras. In particular, we have studied the structure of categories of the triplet W-algebras $\mathcal{W}_{p_+,p_-}$ and $\mathcal{SW}(m)$ defined defined by Feigin, Gainutdinov, Semikhatov and Tipunin and by Adamović and Milas, respectively. We investigate the structure of the projective covers of simple modules and the detailed structure of non-semisimple fusion rules. Although defined differently, our previous studies have shown that there is a deep connection between the triplet W-algebras and the $N = 1$-supertriplet W-algebras. We will continue to study the structure of the categories of modules and clarify the relationship between these vertex operator algebras. The details of our future research goals are described below.

As for the triplet W-algebra $\mathcal{W}_{p_+,p_-}$, the structure of projective covers of the minimal simple modules and the structure of the braided tensor category have not been determined yet. These problems are caused by the fact that the structure of the Zhu algebra $A(\mathcal{W}_{p_+,p_-})$ has not been determined and that the braided tensor category of $\mathcal{W}_{p_+,p_-}$ is not rigid. For the Zhu algebra $A(\mathcal{W}_{p_+,p_-})$, the simplest case $(p_+, p_-) = (2, 3)$ is obtained to have dimension 38 [1], but this has not been determined for the general case. In future studies, we will first try to determine the structure of projective covers of the minimal simple modules of the triplet W-algebra $\mathcal{W}_{p_+,p_-}$ and the structure of the braided tensor category in the case $(p_+, p_-) = (2, 3)$, that is, the central charge is $c = 0$. Since there is only one minimal simple module when the central charge is $c = 0$, it is assumed that the structure of the projective covers of the minimal simple module can be uniquely determined by using the dimensional theorem of the Zhu algebra and the structure of the projective covers of other simple modules.

As for the supertriplet W-algebra $\mathcal{SW}(m)$, we have obtained the interesting correspondence that the non-semisimple fusion ring of $\mathcal{SW}(m)$ is obtained by specializing the non-semisimple fusion ring of $\mathcal{W}_p$ $(p = 2m + 1)$. Here, a non semisimple fusion ring is a commutative ring defined on the set of suitable indecomposable modules. Adamović and Milas obtained the result [2] that the characters of the simple modules of a super triplet W-algebra $\mathcal{SW}(m)$ can be expressed using the characters of the simple modules of the triplet W-algebra $\mathcal{W}_p$. From our result and theirs, we conjecture that there exists an essentially functor of the braided tensor categories from $\mathcal{W}_p$-mod $(p = 2m + 1)$ to $\mathcal{SW}(m)$-mod. The triplet W-algebra $\mathcal{W}_p$ has a hidden sl_2 symmetry and its automorphism group is $\mathrm{Aut}(\mathcal{W}_p) = PSL(2, \mathbb{C})$. We expect that there exists a hidden symmetry like $\mathcal{W}_p$ in the super triplet W-algebra $\mathcal{SW}(m)$, and we would like to clarify the relation between the two triplet W-algebras $\mathcal{SW}(m)$ and $\mathcal{W}_p$ from the aspects of braided tensor category and the automorphism group of vertex operator algebras.

Just as the triplet W-algebra $\mathcal{W}_p$ has a super analogue $\mathcal{SW}(m)$, the vertex operator super algebra $\mathcal{SW}(p, q)$, which is a super analogue of the triplet W-algebra $\mathcal{W}_{p_+,p_-}$, is definable. It was constructed by Adamovi'c and Milas[4], but it is not yet known whether it satisfies C_2-cofinite condition and the classification of simple modules remains unsolved. In future work, we will show that the super triplet W-algebra $\mathcal{SW}(p, q)$ satisfies C_2-cofinite condition and classify the simple modules. In the classification of simple modules, the theory of admissible Jack polynomials[5] is considered to be important.

References

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