Background of Research

We have studied the structure of the categories of modules of the family of vertex operator algebras called triplet W-algebras. In particular, we have studied the structure of the module category of the triplet W-algebra $\mathcal{W}_{p_+,p_-}$ defined by Feigin, Gainutdinov, Semikhatov, Tipunin [1] and the supertriplet W-algebra $\mathcal{SW}(m)$ defined by Adamović and Milas [2]. The triplet W-algebra is one of the few examples of non-rational vertex operator algebras satisfying the C_2-cofinite condition. In general, for rational vertex operator algebras, the category of modules is semisimple, but for non-rational vertex operator algebras, the category of modules is not semisimple, and logarithmic modules appear, where the logarithmic module is a indecomposable module which has L_0 nilpotent rank two or more. Furthermore, if the C_2-cofinite condition is satisfied, the number of simple modules is finite, and the category of modules contains the structure of the braided tensor category constructed by Huang, Lepowsky, and Zhang [3]. Thus, the triplet W-algebras has a good property among the non-rational vertex operator algebras because it satisfies the C_2-cofinite condition, but the concrete aspects of the triplet W-algebra $\mathcal{W}_{p_+,p_-}$, such as the structure of the logarithmic modules and the tensor product between logarithmic modules, are still not fully understood.

Results of Research

We will discuss some our results on the triplet W-algebras $\mathcal{W}_{p_+,p_-}$ and $\mathcal{SW}(m)$.

The results for $\mathcal{W}_{p_+,p_-}$ can be roughly divided into the following two categories.

(1) The structure of the projective covers of all simple $\mathcal{W}_{p_+,p_-}$-modules except the minimal simple modules are determined, where the minimal simple modules are $\mathcal{W}_{p_+,p_-}$-simple modules, which are also Virasoro's simple modules. The structure of these logarithmic modules was conjectured by Gaberdiel, Runkel, and Wood [4] in the case $(p_+, p_-) = (2, 3)$.

(2) We gave a proof of the non-semisimple fusion rules conjectured by Rasmussen [5] and Gaberdiel, Runkel, Wood [4], and determined the structure of the non-semisimple fusion ring, where a non semisimple fusion ring is a commutative ring defined on the set of certain indecomposable modules. By specializing this fusion ring appropriately, we obtain two non semisimple fusion rings of $\mathcal{W}_{p_+}$ and $\mathcal{W}_{p_-}$.

For the supertriplet W-algebra $\mathcal{SW}(m)$, we were able to completely determine the structure of the abelian and braided tensor category over $\mathcal{SW}(m)$-modules. It is shown that the non-semisimple fusion ring of $\mathcal{SW}(m)$ can be obtained from the non-semisimple fusion ring of the triplet algebra $\mathcal{W}_{2m+1}$ by specializing a certain variable. Adamović and Milas showed that the characters of the simple $\mathcal{SW}(m)$-modules is represented by the characters of the simple $\mathcal{W}_{2m+1}$-modules and conjectured that there is a deep connection between the two vertex operator algebras $\mathcal{SW}(m)$ and $\mathcal{W}_{2m+1}$. Our results strongly supports their conjecture.

References

[1] B.L. Feigin, A.M. Gainutdinov, A.M. Semikhatov, and I. Yu Tipunin, Nuclear Phys. B 757(2006),303-343.

[2] D. Adamović and A. Milas, Commun. Math. Phys. **288** (2009), 225-270.

[3] Y. Z. Huang, J. Lepowsky and L. Zhang, arXiv:0710.2687v3 [math.QA].

[4] M. Gaberdiel, I. Runkel and S. Wood, J.Phys. **A42** (2009) 325403.

[5] J. Rasmussen, Nucl. Phys. B **807** (2009) 495.

[6] A. Tsuchiya, S. Wood, J. Phys. A **46** (2013), no. 44, 445203, 40 pp.