Research plan

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I will continue to study homogeneous convex domain, in particular homogeneous cones from various points of view. Homogeneous cones are typical examples of homogeneous spaces of solvable Lie groups so that I focus on differences between reductive and non-reductive homogeneous spaces. As in my paper [1] and my affiliation, I also study on machine learning and the theory of random matrices and continue these topic intensively.

(a) Studies on zeta functions associated with homogeneous cones. I will continue the study (1) on zeta functions associated with homogeneous cones, in particular (i) the problems on their analytic continuity, and (ii) the problems on the gamma matrices (the coefficients of their functional equations). On the problem (i), it is known that zeta functions associated with reductive prehomogeneous vector spaces can be analytically continued to the whole space by the general theory by F. Sato(1982); however, it is still an open problem for non-reductive case. On the problem (ii), I have proved that the gamma matrices can be decomposed into a product of variable-wise matrices [13], and so next I try to generalize a result by Ben Saïd–Clerc–Koufany (2018).

(b) Researches on the rings of invariant differential operators on homogeneous cones. I will continue the research (4) on the rings of invariant differential operators on homogeneous cones, in particular, (i) making explicit calculations on homogeneous spaces which differ from homogeneous cones, and (ii) the problem on expanding the acting groups to the full linear automorphism groups of homogeneous cones. On (i), applying the method of my paper [2], we can give an affirmative answer to the conjecture on the formula of b-function on a certain prehomogeous vector space posed by Ishi–Kogiso (2016), as in the study (4). By calculating other homogeneous spaces or prehomogeneous vector spaces, we can find out new phenomenon. On the problem (ii), in the case of symmetric cones, the rings of invariant differential operators with respect to the full linear automorphism groups, which include those strictly used in my paper [2], play an important role in the analysis on symmetric cones. Note that the groups used in my paper are subgroups of the full linear automorphism groups. It is an important problem to find what kind of phenomenon occur if we remove the assumption of symmetricity.

(c) **Studies on polynomials satisfying local functional equations**

Basic relative invariants of regular prehomogeneous vector spaces admit local functional equation, that is, Fourier transforms of them can be described as products of powers of some polynomials. In order that polynomials satisfy local functional equations, big group actions is not necessary and it is known that there are polynomials satisfying local functional equations which are not basic relative invariants of any prehomogeneous vector space. Therefore, together with professor Takeyoshi Kogiso (Josai University), we study problems what kind of polynomials satisfy local functional equations, or what kind of polynomials can be basic relative invariants of some prehomogenous vector spaces. At present, we have a result on polynomials associated with triangle arrangements (arXiv:2210.10467, [11]).

(d) Studies on random matrices related to graphical models. I will continue the joint work with professor Graczyk on random matrices. In particular, we focus on random matrices related to graphical models. This is because we can define the eigenvalues naturally on a class of homogeneous cones related to graphical models. First, we make simulations of eigenvalue distributions of several Wishart matrices related to graphical models by using computer application. Then, we extract good examples among them and investigate in detail by using the method which is used in our paper [1]. It contributes to several areas like mathematical statistics, theory of random matrices and wireless communications.

(e) Studies on machine learning using invariance/equivariance of group actions Recently, group actions attract attention in communities of machine learning since CNN (convolutional neural network) has a great success. However, it is still unclear "what does mean that AI learns a group action?", and I will tackle this problem together with professor Kenji Fukumizu (ISM). To do so, I use VAE (Variational Auto Encoder) structure for learning group actions.