Summary of research results

I have studied algebraic geometry, especially the moduli spaces of vector bundles on the projective plane $\mathbf{P}^2$. I have studied this moduli spaces using representation theory of quiver. Quiver is a directed graph. We described the birational transformation of the moduli spaces, calculated integrals over the moduli spaces, and also studied derived categories on toric surfaces in collaboration with Hokuto Uehara.

In particular, the integrals over moduli spaces are called the Nekrasov partition functions, which is a generating function counting Young diagrams. More precisely, it is a generating function whose coefficient is a rational function determined by Young diagrams. Recently, I am interested in the relationship among this partition function, integrable systems, and representation theory. Also, as a slightly different topic, in collaboration with Nobuo Hara. We considered a certain finiteness of the singularity over fields of positive characteristics. In the following, we will give a rough explanation of the research process.

Using the concept of stability conditions on derived categories introduced in string theory, we gave natural isomorphisms from moduli spaces of semi-stable sheaves on the projective plane $\mathbf{P}^2$ to moduli spaces of quiver representations. Then we naturally obtain some moduli spaces and morphisms among them. Using this phenomenon called wall crossing, we analyzed the flip structure of the moduli spaces.

After that, in order to calculate integrals over the moduli spaces, we investigated framed vector bundles on $\mathbf{P}^2$. Here, we applied the wall-crossing formula developed by Takuro Mochizuki to obtain functional equations among the Nekrasov partition functions. We also gave a positive solution to the conjecture by physicists Ito-Maruyoshi-Okuda concerning similar functional equations.

I submitted a preprint with Yutaka Yoshida on calculations for handsaw quiver varieties of type A_1. Furthermore, I applied wall-crossing formula to more basic flag manifolds. I also summarized general formula for moduli of framed quiver representations. The following are some of my papers.

R. Ohkawa and H. Uehara, Frobenius morphisms and derived categories on two-dimensional toric stacks, Adv. Math. 244 (2013), 241-267.

We explicitly constructed generators for derived categories of two-dimensional toric Deligne-Mumford stacks. For the natural number m, takeing *m* power of coordinates gives endomorphisms of toric Deligne-Mumford stacks called Frobenius morphisms. We showed that push-forward of structure sheaves by Frobenius morphisms generates derived categories of coherent sheaves on two-dimensional toric Deligne-Mumford stacks.

R. Ohkawa, Flips of moduli of stable torsion free sheaves with c_1 = 1 on $\mathbf{P}^2$, Bull. Soc. Math. France, 142, no 3 (2014), 349–378.

I studied on the structure of birational transformation of moduli spaces of torsion free sheaves on $\mathbf{P}^2$. I applied the fact that the stability condition has one-dimensional degree of freedom by describing torsion free sheaves in terms of quiver representations. This gives us diagrams among moduli spaces. By utilizing this phenomenon called wall crossing, we analyzed flip structure of moduli spaces.

R. Ohkawa, Wall-crossing between stable and co-stable ADHM data, Lett. Math. Phys. 108 (2018), 1485-1523.

We investigated the moduli spaces of framed vector bundles on $\mathbf{P}^2$. By applying the wall-crossing formula developed by Takuro Mochizuki, we obtained functional equations of the Nekrasov partition functions. This functional equation is similar to a conjecture by physicists Ito-Maruyoshi-Okuda. In the following paper, we solved this conjecture in the same way.

R. Ohkawa, Functional equations of Nekrasov functions proposed by Ito-Maruyoshi-Okuda, Moscow Math. J. 20 (2020), no. 3, 531 – 573.

We proved functional equation of Nekrasov partition functions for A_1 singularity. It was predicted by physicists Ito-Maruyoshi-Okuda. More recently, we checked that (-2) blow-up formula can be shown in the same way. We also would like to show another proof of the functional equations for the Painlvé τ function shown by Bershtein and Shchechkin.