今後の研究計画（小俣正朗）

Like in the research results, I am studying obstacle problems related to the motion of bubbles or droplet with touching obstacle or bubbles touches another bubble. The goal is to establish mathematical and numerical method.

Physical image is droplet motion on the obstacle or bubble one the water surface.

Droplet on the celling

multi-bubble on a plane

We call the edge of the set which touches another object is called the junction. On the junction part, energy is expected to be concentrated and is an interesting target for analysis. We will assume interaction between objects are friction and adherence. Moreover, we can set global constraint conditions such as volume conservation, potential energy by the gravity. In this setting, variational methods works better than differential equation treatment. We will apply this for hyperbolic problems.

In detail, we will treat typical potential $I(u) = \int \left(|\nabla u|^2 + \chi_{\{u>0\}}\right)dx$ for describing equilibrium state of membrane peeling problem. The second term is the characteristic function χ_E for the set E.

We will introduce action, it can be written $I = S_{Shape} + U_{Potential}$ for describing kinetics of membrane.

The Lagrangian will be $L = S_{Shape} + U_{Potential} - K_{Kinetic}$, then calculating the first variation, we get the information of this phenomena. If we can calculate the first variation for the characteristic function, moreover, it would be Radon measure, like $\delta_{\partial\{u>0\}}$, the equation will be

$$\chi_{\overline{\{u>0\}}} u_{tt} = \Delta u - \delta_{\partial\{u>0\}} + \int (u u_{tt} + |\nabla u|^2)dx$$

This is one of the targets. Partial results have been obtained in our previous research. Here, we introduce our main tool so called Discrete Morse Flow, i.e. using time semi discretized functional:

$$J_n(u) := \int \frac{|u - 2u_{n-1} + u_{n-2}|^2}{2h^2}dx + I(u) \quad n = 1,2,\cdots.$$

We have an idea to verify this into another style functionals to guarantee energy conversation law. These verifications are also interesting.