Results of research

Takeshi Oota

We have been studying the properties of the matrix models which are closely related to the supersymmetric gauge theories and obtained the following results.

1. We have argued that the level-1 elliptic algebra $U_{q,p}(\widehat{\mathfrak{g}})$ is a dynamical symmetry in the correspondence between 2d field theories and 5d supersymmetric gauge theories. A level-1 $U_{q,p}(\widehat{\mathfrak{sl}}(2))$ module can be realized by an elliptic version of the Frenkel-Kac construction. In a r-th root of unity limit of the deformation parameter p, the $\mathbb{Z}_r$-parafermions and a free boson appear. And the 2d/5d correspondence goes to the correspondence between the 2d coset CFT with para-Virasoro symmetry and 4d $\mathcal{N} = 2$ $SU(2)$ gauge theory on $\mathbb{R}^4/\mathbb{Z}_r$ ([37] of Publication List).

2. We have argued that a unitary matrix model, which is an extension of the Gross-Witten-Wadia model by the logarithmic potential, is closely related to the $\mathcal{N} = 2$ supersymmetric $SU(2)$ gauge theory with two matter hypermultiplets. The spectral curve of the matrix model is isomorphic to the Seiberg-Witten curve of the gauge theory. Using the method of orthogonal polynomials, it is shown that the partition function is a tau function of the Painlevé equation. The double scaling limit of the matrix model corresponds to the limit of the gauge theory to the Argyres-Douglas superconformal fixed point ([38, 39, 40]).

3. We have generalized the above gauge theory/matrix model correspondence to more general case ([41]). It is the correspondence between certain unitary matrix model and the four-dimensional $\mathcal{N} = 2$ gauge theory called the $\hat{A}_{2k,2k}$ theory. We have shown that this unitary matrix model at the k-th multi-critical point corresponds to the (A_1, A_{4k-1}) Argyres-Douglas point of the $\hat{A}_{2k,2k}$ theory.

4. We have studied the multi-critical unitary models in the large-N limit by using the saddle point method. The density function describes how the eigenvalues of the unitary matrix distribute on the unit circle. It is determined by solving the saddle point equation. We have shown that these models have three phases by investigating the properties of the potentials and the density functions. One is a strong-coupling phase, the remaining two are weak-coupling phases. In the strong-coupling phase, the eigenvalues distribute on the whole unit circle without a gap, while in the weak-coupling phases, the eigenvalues distributions develop a gap. We determined the explicit large-N forms of the free energy and the Wilson loops in all three phases. All the phase transitions between the strong/weak phases are shown to be third-order by investigating the free energy. It turns out that one phase transition point is a multi-critical point, while the other is not a multi-critical point. We have also investigated perturbations of the multi-critical unitary matrix models and their double scaling limits ([42]).

5. For the β-deformed A_{n-1} quiver matrix model, a sequence of massive scaling limits of its partition function is taken and an integral representation of $su(n)$ irregular block is obtained. By investigating the pameter region with the maximal symmetry, and through the correspondence between the spectral curve of the matrix model and the Seiberg-Witten curve, the defining condition of the Argyres-Douglas critical hypersurface ([43,44]).