

## Research Plan

I usually have frequent communications on research works with a foreign joint worker Dariusz Partyka (The John Paul II Catholic University of Lublin, Poland). Moreover, I have good communications on research works with David Kalaj (University of Montenegro, Montenegro), Jianfeng Zhu (Huaqiao University, P.R.China) and so on who are studying similar research subjects. If there will be appropriate chances, then I shall visit some of them or attend to some international conference and give our talk on results of our researches. Moreover, I shall attend to the workshop “Prospects of Theory of Riemann Surfaces” which has been held in Yamaguchi University in every year from 2011 and shall be held in Aichi Institute of Technology from 2023, where we could have research communications actively on conformal mappings, quasiconformal mappings, harmonic mappings and so on with 10~12 regular attendances. Concretely I shall have plans of researches such as following (1), (2) and (3).

(1) The projection of a minimal surface which is expressed in terms of isothermal parameters onto the base plane gives an injective harmonic mapping. Conversely, if an injective harmonic mapping satisfies some condition, then it is lifted into a minimal surface. I and Fujino (Nagoya University) shall investigate the relation of respective progress of the research of injective harmonic mappings and the research of minimal surfaces and consider applications. Further, a deformation of harmonic mappings by one parameter family was treated in “D. Kalaj, Quasiconformal harmonic mappings and close to convex domains, *Filomat* 24(2010), no.1, 63-68.” In “A simple deformation of quasiconformal harmonic mappings in the unit disk, *Ann. Acad. Sci. Fenn. Math.* 37(2012), 539-556”, I and the joint worker D. Partyka paid attention to the fact that the deformation was viewed as a holomorphic motion which plays important roles in the theory of quasiconformal mappings. And on the way to discuss some properties of the deformation, we saw that we could develop the same arguments by generalizing convex domains into  $\alpha$ -convex domains. In particular we obtained a result (the injectivity of the holomorphic part of a sense-preserving injective harmonic mapping onto a convex domain) by “Clunie and Sheil-Small, Harmonic univalent functions, *Ann. Acad. Sci. Fenn. Math.* 9(1984), 3-25”, as a corollary. From the view point of quasiconformality of harmonic mappings, the proof was completely different from Clunie and Sheil-Small, and the relation between a sense-reserving injective harmonic mapping  $F$  and the injective holomorphic part  $H$  of  $F$  was investigated. By considering results and methods developed there we

shall try to consider the following new problem, that is, to find some conditions which produce injective harmonic mappings from injective holomorphic functions. Moreover, if the range is generalized into a horizontally convex domain and so on, then we shall try to consider whether the holomorphic part of a sense-preserving injective harmonic mapping is injective or not and investigate to apply such considerations to our research of a deformation of quasiconformal harmonic mappings.

(2) In the research of quasiconformal harmonic mappings, it is known that the conjugate function (the Hilbert transformation) and the Cauchy singular integral of *a.e.* derivative  $f'$  of a quasisymmetric function  $f$  have intimate relations to some properties of the harmonic extension  $F$  of  $f$ . We shall try to investigate quasiconformality and some properties of the harmonic extension by classifying a quasisymmetric function  $f$  in terms of some properties of the Hilbert transformation and the Cauchy singular integral of *a.e.* derivative  $f'$ . This method corresponds to the consideration by classifying a quasisymmetric function  $f$  by Hardy spaces and so on where the derivative of the holomorphic part of  $F$  and the derivative of the conjugate of the anti-holomorphic part of  $F$  belong.

(3) Jointly with D. Partyka I have obtained many results for quasiconformal and harmonic mappings (with respect to the Euclidean metric). We shall then try to obtain following sort of results. Although we obtained Heinz type inequalities in the case where the range is a bounded convex domain, we shall then consider that to what extent the method would be effective in the case where the range is a Jordan domain with the rectifiable boundary curve or an  $\alpha$ -convex domain. Furthermore, we shall try to consider to give characterizations of quasiconformality of harmonic mappings even in the case where the range is a Jordan domain with the rectifiable boundary curve, and investigate applications to the research of holomorphic motions which play important roles in the theory of quasiconformal mappings. Moreover, we shall try to consider to what extent analogous results hold for quasiconformal and hyperbolically harmonic mappings (that is, harmonic mappings with respect to the hyperbolic metric). Further, for harmonic mappings with respect to several metrics related to holomorphic quadratic differentials which appear in the theory of Gerstenhaber-Rauch principle and so on in the variational problem of Douglas-Dirichlet integrals, we shall try to consider to what extent analogous results hold.