今後の研究計画の英訳

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Hessenberg varieties are important subvarieties of a flag variety. It depends on an element of the Lie algebra and a good subset of the positive root system. This good subset is called a lower ideal. If this element of the Lie algebra is regular, nilpotent, or semisimple, the Hessenberg variety is called so. As I said in the results of my research, I determined the cohomology rings of regular nilpotent Hessenberg varieties explicitly. However an explicit description of the cohomology ring of a regular semisimple Hessenberg variety is not known in general. Regular semisimple Hessenberg varieties, unlike regular nilpotent Hessenberg varieties, are not smooth, but they have an action of the maximal torus and are GKM spaces. So the restriction map of equivariant cohomology rings to the fixed point set is injective. Therefore one can calculate their equivariant cohomology rings in theory, but the calculation is difficult.

Now I am researching with Prof. Masuda the cohomology rings of regular semisimple Hessenberg varieties. We determined them in the case of the rings are generated by elements of degree 2. As an improvement of this research, I will study the cohomology rings of regular semisimple Hessenberg varieties in the case of the rings are generated by elements of degree 4 or less, of degree 6 or less, and so on. Finally I will determine an explicit description of the cohomology rings of any regular semisimple Hessenberg varieties. Their generators and relations are not known, but I will find them with assistance of computers in the case of low dimension. And the results of the current research will help me to find them.

As other research object, for a regular semisimple Hessenberg variety of type A, there is a special manifold which is called its Hessenberg twin. They are born like twins from a subspace of $U(n)$. Their difference comes from the left and right multiplication of a maximal torus of $U(n)$. So their torus equivariant cohomology rings are isomorphic. The complex coefficient cohomology ring of a Hessenberg twin has a different action of symmetric group from the usual one, and as a non-graded representation it is a regular representation. To determine how many times each irreducible representation appear in which degree is an interesting problem, and it was solved in the case of a flag manifold. Some combinatorial numbers of Young tableaux give the answer. I think that this number must be interpreted as counting of roots of the Weyl group and that this counting with the lower ideal gives the answer in the case of Hessenberg twins combinatorially. Actually I verified it works well in low dimensions.

Regular semisimple Hessenberg varieties are GKM spaces, and elements of their cohomology rings are described explicitly by assigning a polynomial to each fixed point. This way of presentations of cohomology rings is called GKM theory. By GKM theory we can understand geometry of regular semisimple Hessenberg varieties as combinatorics of the fixed point sets, and we can also make concrete calculations. By the way, the cohomologies of regular semisimple Hessenberg varieties correspond to the chromatic symmetric functions. I will reveal geometric meaning of chromatic symmetric functions more directly by GKM theory. I will also reveal geometric meaning of LLT polynomials through the twins.