# これまでの研究成果の英訳

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**(1) Motivation of my research**

Flag maniflods have high symmetry and their geometric property is often described in terms of combinatorics. For example, the rational cohomology ring of a flag manifold is the coinvariant ring of the corresponding Weyl group, the poset structure of the Weyl group determines the Bruhat decomposition of the flag manifold, and the complex dimension of the cell is equal to the length of the corresponding element of the Weyl group. The aim of my research is to reveal the relation of combinatorics and other geometrical objects.

**(2) Results of my research**

Let $G$ be a compact connected Lie group and $T$ be a maximal torus of $G$. The maximal torus $T$ acts on the flag manifold $G/T$ by the left multiplication. For $G$ is $F_4$ or $E_6$, I determined the $T$-equivariant cohomology ring with integer coefficients of the flag manifold $G/T$ as the quotient ring of the polynomial ring by the ideal generated by explicit elements.

I also determined the $T$-equivariant cohomology ring with integer coefficients of the flag manifold of type C. I employed the GKM theory to determine these cohomology rings. The GKM theory for flag varieties is understood well in the point of view of the Bruhat decomposition, and it describes the attaching maps in terms of the root system. My calculation was based on the combinatorial structure, so I gave geometrical meaning to my representation of the cohomology ring.

Hessenberg varieties are subvarieties of the flag variety $G/T$. Let $\Phi^+$ be the positive root system of $G$. A Hessenberg variety is determined by two data; an element of the Lie algebra of $G$ and a "good" subset of $\Phi^+$ （this is called a lower ideal）. When $I = \Phi^+$, the Hessenberg variety is the flag variety. It was known that Hessenberg varieties includes Springer varieties, so Hessenberg varieties are important. I determined, with other researchers, that the cohomology ring of a regular nilpotent Hessenberg variety as a quotient ring of a polynomial ring in terms of a lower ideal. Moreover we proved that the ring of the Weyl group invariant elements of the cohomology ring of a regular semisimple Hessenberg variety is isomorphic to the cohomology ring of the regular nilpotent Hessenberg variety. These results also indicate that the combinatorics of the hyperplane arrangements associated with the lower ideal appears as the geometry of the regular nilpotent Hessenberg variety.

I gave a characterization of the cohomology ring of a regular semisimple Hessenberg variety of type A being generated in degree 2, and I determined its generators and relations.

For a regular semisimple Hessenberg variety of type A, there is a smooth manifold called its twin. There is an action of symmetric group on the cohomology of the twin, and I showed that it is described by the LLT polynomial corresponding to the lower ideal.

There are special rings which are called the double coinvariant ring of pseudo-reflection groups. They are analogues of the equivariant cohomology rings of flag varieties. I described it as a quotient ring of a polynomial ring in terms of pseudo-reflections by applying some method using in combinatorial algebra.