Future research plan

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• **Optimal L^2(mass)-decay for dissipative nonlinear Schrödinger equations:**

On the dissipative nonlinear Schrödinger equations, the relation between spatial regularity and time mass decay of solutions was revealed by previous works, however it does not be revealed a sufficient condition for solutions with the optimal mass decay. Kim-Sunagawa (2014) showed the optimal mass decay of solutions for nonlinear dissipative Klein-Gordon equation by the finite propagation of solutions. On the other hand, the infinite propagation for nonlinear Schrödinger equations cause a difficulty to analysis obtaining the optimality of the mass decay under we consider the Cauchy problem. Our aim is to show that the order $(\log t)^{-\frac{1}{2}}(\log\log t)^{\frac{1}{2}}$ is sharp for the related Cauchy problem. To this end, we carefully study regularity and the shape of solutions by analyzing method for approximate solutions in Li-Nishii-Sagawa-Sunagawa (2022) and using argument for the Pseudo conformal transformation in Cazenave-Han-Naumkin (2021). If the aim is shown, then it may clarify the characteristic mechanism of dissipative and dispersive nature in the nonlinear Schrödinger equation.

• **Analysis of a dissipative nonlinear Schrödinger system:**

For the nonlinear Schrödinger system, the gauge invariance on nonlinearities cause the resonance of amplitude and phase for the wave functions. This phenomenon is the nonlinear effect and important to consider the asymptotic behavior of solutions to quadratic nonlinear Schrödinger equations with nonlinear dissipation (Li-Nishii-Sagawa-Sunagawa (2021) treated a cubic nonlinear Schrödinger system). Our aim is to analysis the dissipative nature from the quadratic nonlinear Schrödinger system describing the Raman effect. To this end, we treat the system with complex nonlinear coefficients. Under this complex condition, the L^2-norm (mass) of solutions no longer conserve and loss of monotonicity in time valuable. Hence the asymptotic behavior of solutions to this system is complicated and we need to have the sharp analysis on approximate solution given by the associated nonlinear ODE system by using power series expansion for the solution exhibiting real analyticity in space valuable.

• **The relation between dissipative nonlinear Schrödinger equations and physical phenomenon:**

It is known that the Korteweg-Euler equation is derived from the nonlinear Schrödinger equation which has mass conservation law by applying the Madelung transform. The Korteweg-Euler equation describes a quantum fluid mechanism related to the Bose-Einstein condensate which does not hold dissipative nature from the particle friction. We study asymptotic behavior on a quantum fluid model with friction between particles by studying Korteweg-Navier-Stokes equation derived from a dissipative nonlinear Schrödinger equation.