Research results

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The following number [·]... correspond to the one in the list on research achievement.

•Analysis of solutions to dissipative nonlinear Schrödinger equations ([1], [3], [4], [7], [8]):

We clarify the existence of a critical exponent of nonlinearities for dividing an asymptotic behavior of solutions to dissipative nonlinear Schrödinger equations. This exponent is the threshold that the mass (L^2-norm) of solutions decays or not and this one corresponds to the Fujita-exponent which is appeared in the nonlinear heat equations. The exponent also corresponds to the Barab-Ozawa critical exponent which decide the scattering situation on nonlinear Schrödinger equations under the mass conserved setting. In [4], we show the mass of solutions does not decay when the nonlinear power surpass the critical one while the mass decays if the nonlinear power is less or equal to the critical power. In this situation we call the power as subcritical or critical exponent. We also show in [1],[3],[7] that a relation between spatial regularity of solutions and a time decay rate of its mass and we lead to almost optimal decay rate when the solution have spatial analyticity. The following table implies a differential order and the time decay rate of solutions to dissipative nonlinear Schrödinger equations in one space dimension with the critical exponent.

Table 1: The relation between regularity and mass dacay

spatial regularity	$u \in H^k(\mathbb{R})(k \geq 1)$	$u \in G_v^s(\mathbb{R})$	$u \in G_v^1(\mathbb{R}) = C^\omega$
decay rate	$(\log t)^{-\frac{1}{2}+\frac{1}{2(2k+1)}}$	$(\log t)^{-\frac{1}{2}}(\log\log t)^{\frac{s}{2}}$	$(\log t)^{-\frac{1}{2}}(\log\log t)^{\frac{1}{2}}$

where $H^k(\mathbb{R})$ denotes the usual Sobolev space and $G_v^s(\mathbb{R})$ stands for the Gevrey class based on the Lebesgue L^2 space with differential index $s \geq 1$. The table 1 implies if solutions belong to $G_v^s(\mathbb{R})$ which ensures that solutions can be infinitely differentiable, then its decay rate shows almost optimal ([1]).

•On the optimal mass decay of dissipative solutions ([5], [6], [10], [11]):

It is difficult to construct smooth solutions for nonlinear Schrödinger equations with subcritical nonlinearity because a lower power nonlinearity does not regular. We consider a final state problem and construct solutions whose top term is given by the solution to the related ordinary differential equation which decays as $(\log t)^{-\frac{1}{2}}$ for $t \to \infty$. We also find that a sufficient condition that solutions show optimal mass decay is that the Fourier transform of the solution has a compact support. This implies the solution has spatial real analyticity. A sufficient condition for the asymptotic data which gives the optimal mass decay of associated solutions is a function like the Friedrichs mollifier, whose support is compact in frequency part.