

January 5, 2023
Ryutaro Sekigawa

Future research plan

I plan following studies:

1. Constructing “new polynomials” and studying the properties, including monogeneity
2. Providing an alternative proof of Coleman formula and its applications

1. Constructing “new polynomials” and studying the properties, including monogeneity

The objective is to obtain a general method in research of power integral bases. The basic method used in previous research has the problem that it is not general due to its limitations on degree and conditions for application. Therefore, I aim to obtain many concrete results and to develop prospects for formulation.

The main method is to construct and organize new polynomials by descent Kummer theory, based on the known cyclic polynomials used in research on monogeneity.

I proved that there exist infinitely many monogenic cyclic extensions using Rikuna polynomial. It is known that Rikuna polynomial can be interpreted as a descent Kummer theory of degree 2. On the other hand, the descent Kummer theory is formulated in the specific degrees by using an algebraic torus and is specifically computable. Using the descent Kummer theory, I construct new polynomials with properties similar to Rikuna polynomials. Next, I characterize whether the splitting fields have a power integral basis, based on previous research. I am working on this research with Kazuki Kanai and have discovered a family of polynomials containing the known important polynomials.

In addition, I study the properties of the constructed polynomials, based on Shanks polynomial. Shanks polynomial is the basic case of Rikuna polynomial and is well known for the class number and the regulator.

2. Providing an alternative proof of Coleman formula and its applications

The objective is to provide an alternative proof for Coleman formula that has a prospect of a generalization. Coleman calculated the absolute Frobenius on Fermat curves explicitly. This is an important result related to Gross-Koblitz formula, Kashio-Yoshida conjecture, and Stark conjecture. We aim for an alternative proof that overcomes the problem of the need for a concrete basis in Coleman's method for a generalization and its applications. This work is joint work with Tomokazu Kashio and Hyuga Yoshizaki.

The main tool of the proof is functional equations and continuity. Kashio considered both sides of Coleman formula as functions and showed that a continuity implies a large part of the formula. Building on his results, we prove the continuity and elucidate the necessary functional equations and provide an alternative proof.