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Summary of research results

【Positioning of the research】

I have studied algebraic number theory especially monogenity. I have studied the classical problem of characterizing algebraic number fields whose rings of integers have a power integral basis. This research is a study of invariants associated with algebraic number fields.

【Main research results】

The basic method used in previous research on power integral bases has the problem that it is not general due to its limitations on degree and conditions for application. It is necessary to obtain many concrete results for the development of research. I focused on generic polynomials of cyclic extensions, which can be defined independent of the degree. Starting with research on cyclic cubic fields, which is the basic case, I studied cyclic extensions of odd prime degree.

The characterization of cyclic cubic fields with power integral bases

Using Shanks polynomial, which is a generic polynomial, we provide the equivalent conditions for the monogenity of any cyclic cubic field. The strength is that it is more explicit and easier to judge than the conditions of previous research. We use the first Galois cohomology group of the unit group in the proof, which is not used in previous research.

1. T. Kashio, R. Sekigawa, The characterization of cyclic cubic fields with power integral bases, Kodai Math. J. 44 (2021), no. 2, 290-306.

Infinite family of monogenic cyclic extensions of odd prime degree

Using Rikuna polynomial, which is a generalization of the Shanks polynomial, I have studied the monogenity of cyclic extensions of odd prime degree and obtained two main results. The first is an explicit sufficient condition for the monogenity. The strength of this condition is its versatility: it does not depend on the degree size. Second, I proved that there exist infinitely many monogenic cyclic extensions. Previous research suggests that monogenic fields is rare. I believe that this result is a breakthrough in research on monogenity, obtained by using generic polynomial. The proof is also unique. Using Shintani's fundamental domain, I returned an ideal counting problem to an element counting problem.

2. R. Sekigawa, Rikuna's generic cyclic polynomial and the monogenity, J. Number Theory 231 (2022), 239-250.