Research plan

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The drift-diffusion equation possesses the mass conservation law and the entropy dissipation structure. For the two-dimensional semilinear case, it is well-known that 8π is a threshold of the total mass between the global existence and the finite time blow-up for positive solutions to the problem. When the total mass of the initial data is bigger than 8π, the maximum value of the solution to the problem diverges to infinity in a finite time at some singular point. On the other hand, if the initial condition is less than or equal to the threshold, it is known that the solution exists until time infinity. However, the threshold constant for the behavior of a solution to the problem is unknown.

This research aims to

(1) The identification of the threshold to classify the behavior of solutions to the Cauchy problem of the drift-diffusion equation.

To determine the threshold, we need to verify the compactness of the function sequence consisting of solutions to the problem. We apply the profile decomposition derived in the paper [6] to the Cauchy problem of the degenerate drift-diffusion equation.

(2) The study of the partial regularity of solutions to the problem.

Since the partial regularity of solutions to the drift-diffusion equation can quantitatively estimate the effect of the spatially local integral quantity of solutions to the problem, this is an important property related to both regularity and the behavior of solutions blowing up in a finite time. In the drift-diffusion equation, nonlocal effects appear in nonlinear terms, making it difficult to study the spatially local behavior of the solution to the problem. By refining the estimates derived from the paper [8], we study the partial regularity of solutions to problems.