A summary of my research

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The drift-diffusion equation appears in various mathematical models in astronomy, physics, and chemical biology. This equation is a system of nonlinear partial differential equations including two unknown functions, and one of them is a nonlocal potential given by the Poisson equation. The drift-diffusion equation possesses the mass conservation law and the entropy dissipation structure. The Lyapunov functional of a semilinear drift-diffusion equation corresponds to the Boltzmann–Shannon entropy. The estimates for the Lyapunov functional are useful for studying the decay estimate and asymptotic behavior for solutions to the drift-diffusion equation. The functional inequalities imply the relation between the entropy functional, moment, and potential energy for the probability density functions. In the joint work [1] with Professor Hideo Kubo at Hokkaido University and Professor Takayoshi Ogawa at Tohoku University, we identified the optimal constant and the optimizer of the moment inequality for the probability density functions whose logarithmic moment is bounded. In the paper [4], we showed the moment inequality for the Rényi entropy, a generalized Boltzmann–Shannon entropy. As an application to the nonlinear partial differential equations, we showed the asymptotic behavior of solutions to the Cauchy problem of the drift-diffusion equation in the joint works [3] and [6].

The Cauchy problems of the incompressible Navier–Stokes equation and Fujita type heat equation are well-posed in function spaces containing non-decaying functions, e.g., uniformly local Lebesgue spaces. On the other hand, the well-posedness of the Cauchy problem of the drift-diffusion equation in uniformly local Lebesgue spaces had not been known because of the nonlocal effect of the nonlinear term. In the paper [2], by extending the functional inequality for the potential on the uniformly local Lebesgue space, we proved the well-posedness of the Cauchy problem (see also [7]). Based on this result, we considered the relaxation of the Keller–Segel system, which describes a chemotactic aggregation of microorganisms, on uniformly local Lebesgue spaces in a joint work [5] with Professor Takayoshi Ogawa.