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Research Result

Let $\beta > 1$. Denote by $\lfloor y \rfloor$ the integer part of y and $\{y\}$ the fractional part of y. Define $T: [0,1] \to [0,1)$ by $T(x) = \{\beta x\}$. Then we have the expansion of $x \in [0,1)$, that is,

$$x = c_1\beta^{-1} + c_2\beta^{-2} + \cdots + c_n\beta^{-n} + \cdots = \sum_{n=1}^{\infty} c_n\beta^{-n}$$

where $c_n = \lfloor \beta T^{n-1}(x) \rfloor$. We call such expansion a *beta-expansion* of x and write $d_\beta(x) = c_1c_2\cdots$. When $x \geq 1$, a beta-expansion of x is given by

$$x = c_1\beta^{L(x)-1} + c_2\beta^{L(x)-2} + \cdots + c_{L(x)-1}\beta + c_{L(x)} + c_{L+1}\beta^{-1} + \cdots = \sum_{n=1}^{\infty} c_n\beta^{L(x)-n}$$

where $L(x) = \min\{n \in \mathbb{Z}_{\geq 0} \mid x\beta^{-n} < 1\}$ and $d_\beta(\beta^{-L(x)}x) = c_1c_2\cdots$. We say that x has a finite beta-expansion if its tail is only zero. Denote by $\mathrm{Fin}(\beta)$ the set of real number x such that $|x|$ has a finite beta-expansion.

It is known that any positive integer has a finite decimal expansion. As a generalization of this finiteness property, Frougny and Solomyak introduced the following conditions:

(F_1) $\mathbb{Z} \subset \mathrm{Fin}(\beta)$ (PF) $\mathbb{Z}_{\geq 0}[\beta^{-1}] \subset \mathrm{Fin}(\beta)$ (F) $\mathbb{Z}[\beta^{-1}] \subset \mathrm{Fim}(\beta)$

(1) **Finite $\boldsymbol{\beta}$-expansion and Odometers** ([1] in Peer-reviewed papers)

We introduced the odometer $H_{\beta^{-1}}$ associated with β-numeration system and proved the followings: β has property (F) if and only if $H_{\beta^{-1}}$ is surjective, and β has property (PF) if and only if $H_{\beta^{-1}}$ is injective. Furthermore, when β is an algebraic integer, we can represent a procedure of carry operation in $H_{\beta^{-1}}$ by a transducer. As a result, we also proved that β has property (F) if and only if $H_{\beta^{-1}}$ is computable. This is a joint work with M. Yoshida.

(2) **Some class of cubic Pisot numbers with finiteness property** ([2] in Peer-reviewed papers)

Akiyama characterized cubic Pisot units with property (F). Also, Akiyama et al. found some classes of cubic Pisot numbers with property (F) by using a set of witnesses. In this paper, we obtained a generalization of Akiyama's cubic Pisot units theorem by hand computing. Moreover, in that proof, we found some classes of cubic Pisot numbers with property (F) by using the set smaller than set of witnesses. As a result, we found a new class of cubic Pisot numbers with property (F). This is a joint work with M. Yoshida.

(3) **Finite beta-expansions of natural numbers** (in preparation)

It is known that β is a Pisot number if β has property (F_1). However, other results are not known. In this study, we find a new sufficient condition for property (F_1) and the β which has property (F_1) without (PF).