Research Plan

Kanehisa Takasaki

I will do researches on the following subjects.

Enumerative geometry and integrable hierarchies In the last several years, I have been studying integrable hierarchies related to Gromov-Witten invariants. I want to tackle several issues that have not been resolved. One of them, which has been a main target of this research, is to understand the Dubrovin-Zhang and Givental theories. Dubrovin and Zhang proposed a program to construct an integrable hierarchy for all-genus invariants from an integrable hierarchy for invariants of genus zero (in other words, from a Frobenius manifold). Givental used a bosonic Fock space to present an explicit formula of the generating function of all-genus invariants (which amounts to the τ-function in the sense of Dubrovin and Zhang). Although conceptually distinct from the fermionic formula of the τ-functions of the KP and Toda hierarchies, Givental's formula seems to indicate the presence of a similar integrable structure hidden behind.

Topological string theory and asymptotic analysis Recently, a group of physicists, M. Alim et al., reported an approach to topological string theory on a resolved conifold (a special toric Calabi-Yau threefold) from Borel summation and exact WKB analysis. This seems to be an attempt to reconsider a Riemann-Hilbert problem studied by Bridgeland in the context of the Donaldson-Thomas invariants. These researches may be placed along the line of the work of Kashaev et al. on quantum dilogarithmic functions, and related to the work of Iwaki, Koike and Takei on the τ-function of hypergeometric equations as well. Bearing these results in mind, I will try to generalize them to the so called strip geometry and to relate these issues to isomonodromic deformations.

Isomonodromic deformations Isomonodromic deformations are applied to a wide area of geometry. I am particularly interested in the development that starts from Dubrovin's theory of Frobenius manifolds in the 1990's and reaches Bridgeland's recent studies on BPS structures and Joyce manifolds. Also intriguing is the approach from topological recursion by Iwaki and his collaborators. My work on Whitham modulation equation and Hamiltonian structures in the second half of 1990's seems to have a point of contact with these recent trends. I want to probe a possibility of further research therein.

KP and Toda hierarchies of B, C and D types Variations of the KP hierarchy, roughly classified into the B, C and D types, were introduced in the early 1980's just after the KP hierarchy (which amounts to the A type) was devised. Applications, as well as foundations, of those variants have been studied for years. For example, those variants of KP hierarchies, like the ordinary KP and Toda hierarchies, were applied to random matrix models and orthogonal polynomials. Recently, the KP hierarchy of the B type has been attracting attention in the context of enumerative geometry, in particular, Hurwitz numbers of the Riemann sphere. On the other hand, similar variants of the Toda hierarchy are also known since the 1980's. Recently, however, a completely new type of Toda hierarchy was discovered by Zabrodin and Krichever. Moreover, a new approach from the Kostant-Toda hierarchy was initiated by Kodama and his collaborators. Further exploration of variations of the KP and Toda hierarchies is a very important issue.