Research achievements

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I am specialized in algebraic analysis and mathematical physics. My research has been mostly focussed on the theory and applications of integrable systems.

Foundations of integrable hierarchies Around 1981, Mikio Sato proposed the KP hierarchy and its multi-component generalizations as a unified framework for various soliton equations. Kimio and Ueno devised the Toda hierarchy and its multi-component generalizations as a Toda lattice version thereof. The Toda hierarchy is nowadays recognized as one of the most fundamental integrable hierarchies. Afterwards, aiming at higher dimensional generalizations, I studied the self-dual Yang-Mills equation and the self-dual gravity equation until 1990, then attempted to construct higher dimensional integrable hierarchies based on Moyal algebras until 1994.

Foundations of dispersionless integrable hierarchies Dispersionless integrable hierarchies are obtained from the KP, Toda and other integrable hierarchies by replacing the commutators of differential or difference operators in the Lax formalism with the associated Poisson brackets. Those dispersionless Lax equations resemble the self-dual gravity equation and can be treated by the idea of twistor theory. Takashi Takebe and I started to study the dispersionless KP and Toda hierarchies around 1990, and completed its foundations by 1995. In the beginning of the 2000's when dispersionless integrable hierarchies attracted renewed interests of researchers, we resumed the research and produced several results.

Applications of integrable hierarchies to mathematical physics In the first half of the 1990's. I studied applications of integrable hierarchies to topological conformal field theory and $c = 1$ string theory. Afterwards, Nakatsu and I considered four-dimensional $\mathcal{N} = 2$ supersymmetric gauge theory, and explained the low-energy effective theory (the Seiberg-Witten theory) in the language of the Toda hierarchy and the Whitham modulation equations. I found a similar structure in the blowup formula of topological gauge theory around 1999. Nakatsu and I resumed collaboration in 2004 to study five-dimensional supersymmetric gauge theory. We interpreted its instanton partition function as a statistical model of three-dimensional Young diagrams (the melting crystal model), and discovered that it is a solution of the one-dimensional Toda hierarchy.

Isomonodromic deformations and finite-dimensional integrable systems Motivated by researches on the Seiberg-Witten theory, I moved the main subject of my research to isomonodromic deformations and finite-dimensional integrable systems for several years after the end of the 1990's. As regards isomonodromic deformations, I found a relation to the Whitham modulation equation, isomonodromic deformations on an elliptic curve, the Painlev'e-Calogero correspondence, etc., and also studied Hamiltonian structures. As regards finite-dimensional integrable systems, Ryu Sasaki and I did a series of researches on the Calogero-Moser system and its variations. I considered integrable systems related to elliptic curves and special algebraic surfaces as well.

Integrable hierarchies related to topological string theory and enumerative geometry The melting crystal model is closely related to topological string theory on a three-dimensional toric Calabi-Yau manifolds. The amplitudes of topological string theory of that kind can be described by the method of topological vertex. After 2010, while continuing the research on the melting crystal model, Nakatsu and I studied the topological vertex and the Hogde integrals related to the topological vertex. In recent years, I extended the scope of my research to topics of enumerative geometry, such as the Hurwitz numbers and the Gromov-Witten invariants of the Riemann sphere, and pointed out that the Toda hierarchy and its various reduced systems underlie therein.