RESEARCH RESULTS

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My research interests are representation theory of Lie algebras and applications to other areas in mathematics and physics. Recently, I am interested in the i-quantum groups, which are algebraic systems appearing in the theory of quantum symmetric pairs. A quantum symmetric pair is a quantization of certain pair of complex Lie algebras, called a symmetric pair. To be more specific, it is a pair of a Drinfeld-Jimbo quantum group and its certain coideal subalgebra. Such a subalgebra is referred to as an i-quantum group. My study is more or less based on an idea "The i-quantum groups are generalizations of the quantum groups. Generalize everything from quantum groups to i-quantum groups" (i-program), proposed by Bao-Wang.

Representation theory of the i-quantum group of type AIII. Bao-Wang proved that the i-quantum group of type AIII and the Hecke algebra of type B are in Schur duality. In joint works with them, I extended this result to the unequal parameter case. In particular, this extension covers the asymptotic case. Applying the representation theory of the Hecke algebra of type B with asymptotic parameter via Schur duality, I described the structures of finite-dimensional irreducible modules over the i-quantum group of type AIII in detail. Also, by extending the theory of i-canonical bases due to Bao-Wang, they and I determined the irreducible characters in the Bernstein-Gelfand-Gelfand category of the ortho-symplectic Lie superalgebra completely.

Classical weight modules. I defined the notion of classical weight modules in representation theory of the i-quantum groups by generalizing the notion of weight modules in representation theory of the quantum groups. After developing a general theory of classical weight modules, I classified the finite-dimensional classical weight modules for the i-quantum groups of type AI, AII, and AIII.

Representation theory of the i-quantum groups and combinatorial structures. I introduced the notion of i-crystals by abstracting combinatorial structures appearing in representations of certain quasi-split i-quantum groups. It is a generalization of the notion of crystals appearing in the theory of quantum groups. It turned out that the i-crystals play an essential role in the construction of the crystal bases of the modified i-quantum groups.

Analyzing the finite-dimensional representations of the i-quantum group of type AI by means of i-crystals, I constructed a combinatorial model in terms of Young tableaux. This model is also a new combinatorial model for finite-dimensional representations of the orthogonal Lie algebras.

Applying the theory of i-crystals to Kirillov-Reshetikhin crystals, Kusano, Okado, and I succeeded in constructing combinatorial K-matrices, which play a central role in integrable systems and discrete dynamics.

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