# Research Result

Reiji Yoshioka

## • 2d/4d(5d) correspondence and some limits

The 2d/4d correspondence states the equivalence between the conformal block in 2d conformal field theory and the instanton partition function in 4d $su(n)$ supersymmetric gauge theory. It is also possible to extend the similar connection between 2d CFT based on the $q$-deformed Virasoro algebra and the 5-dimensional supersymmetric gauge theory (2d/5d correspondence), which reduces the original 2d/4d correspondence in the $q \to 1$ limit.

I have studied the $q \to -1$ limit and have shown that the free boson and free fermion which describe the superconformal Virasoro algebra can be obtained from a $q$-deformed boson in this limit. On the 5d side, I have taken the same limit of the 5d instanton partition function and have confirmed that the results are equal to the 4d ALE instanton partition function at the lower level at least. The general root of unities are also discussed, and the emergence of parafermions, etc. was demonstrated.

On the gauge theory side, the number of flavors is $N_f = 2n$ in the original 2d/4d correspondence. By taking the mass infinity limit, the degrees of freedom of the flavors can be decoupled. On the CFT side, the counterpart is known as an irregular conformal block. The $\beta$-deformed matrix model, which is equivalent to the conformal block, converts to a unitary matrix model in the irregular limit. Previous studies have been mostly concerned with $n = 1$ and little attention has been paid to the irregular limit in general $n$, i.e., multi-matrix model with log-type potentials in the irregular limit. I have established the irrgular limit procedure for $N_f = 2n-1, 2n-2$. This extension suggested that the model with general $n$ has a rich structure that does not exist at $n = 1$. As an example, I found the following flavor mass relations that restore maximum discrete symmetry based on the Dynkin diagram at $N_f = 2n-2$:

$$m_i = m_{2n-1-i}, \quad i = 1, \cdots, n-1.$$

The set in parameter space which maximally degenerates the Seiberg-Witten curve is provided by these relations, showing that in general $n$ the set is not a point but a hypersurface.

## • tensor model

The tensor model can be regarded as a generalization of rectangular matrix model to higher rank. Recently, the tensor model is receiving a lot of attention because of its relation to the low dimensional AdS/CFT and quantum gravity. However, unlike ordinary matrix models, the gauge-invariant operators in the tensor model have nontrivial structures, which make it difficult to use ordinary methods such as the Virasoro constraints.

I demonstrated the following Op/FD correspondence between the tensor models of different ranks.

$$\text{Operator (rank } r) \quad \Longleftrightarrow \quad \text{Feynman Diagram (rank } r-1)$$

Each operator in the tensor model is, therefore, labeled with the Feynman diagram. In particular, in the case of rank 3, it is extended to an Op/FD/dessin correspondence including a one-to-one correspondence with graphs called dessins. Here the dessin is a graph consisting of vertices of two colors and edges connecting them embedded on a two-dimensional surface. I succeeded in building a concrete relationship. By using this correspondence, all operators up to level 5 in the rank 3 tensor model were classified according to properties of FD and dessin, for example, the number of vertices. In addition, I have established the interpretation of the cut & join operations as diagrammatic manipulations by expressing them in the language of dessin.