Previous research

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The elliptic sigma functions $\sigma(u)$ and the elliptic functions $\wp(u)$, which are defined and studied by Weierstrass, are generalized to the multivariable sigma functions and the Abelian functions associated with the hyperelliptic curves by Klein and Baker about 100 years ago. In the 1990s, Buchstaber, Enolski, and Leykin generalized the theory of the hyperelliptic sigma functions and the hyperelliptic Abelian functions to a plane curve called (n, s) curve. I extended the theory of the sigma functions and the Abelian functions for the (n, s) curves to telescopic curves, which include the (n, s) curves.

The Abelian function associated with a hyperelliptic curve of genus g is a meromorphic function on $\mathbb{C}^g$ that satisfy $2g$ periodicity conditions on $\mathbb{C}^g$. Baker, Buchstaber, Enolski, and Leykin proved that the Abelian functions associated with hyperelliptic curves satisfy the KdV–equations. For hyperelliptic curves of genus 3, I constructed the theory of the meromorphic functions that satisfy 6 periodicity conditions on the zero set of the sigma functions and derived the partial differential equations integrable by these functions, which is a joint work with V. M. Buchstaber. These partial differential equations are two parametric deformations of the KdV–equations.

In the 19th century, many mathematicians such as Jacobi and Bolza gave many examples of hyperelliptic integrals that can be reduced to elliptic integrals. This problem is closely related to the split Jacobians. This knowledge is applied to the isogeny based cryptography. In [Enolski and Salerno 1996], when a hyperelliptic curve of genus 2 admits a morphism of degree 2 to an elliptic curve, the relationships between the Abelian functions of genus 2 and the Jacobi elliptic functions are derived. I derived the relationships between the Abelian functions of genus 2 and the Weierstrass elliptic functions under the same conditions as [Enolski and Salerno 1996], which is a joint work with V. M. Buchstaber. In integrable systems, when solutions of differential equations are expressed in terms of the Abelian functions associated with an algebraic curve, it is important to express them in terms of the Abelian functions associated with algebraic curves of lower genus. Our results can be applied to integrable systems.

It is important to determine coefficients of the power series expansion of the sigma function. Buchstaber and Leykin derived a method to use the heat equation satisfied by the sigma function. Nakayashiki derived methods to use the algebraic expression of the sigma function or the expression of the sigma function in terms of the tau function. Onishi proved that the coefficients of the power series expansion of the sigma function for the (n, s) curves are included in the ring generated by the coefficients of the defining equation of the curve and $\frac{1}{2}$ over $\mathbb{Z}$. I extended this result to the telescopic curves.

In theory of functions, it is an important problem to decompose a multivariable meromorphic function into a product of two meromorphic functions. Two meromorphic functions on $\mathbb{C}^2$ which give solutions to the inversion problem of the ultra-elliptic integrals are derived by Matsutani and Grant. These meromorphic functions coincide on the zero set of the two-dimensional sigma function. I decomposed the difference of the two meromorphic functions into a product of the sigma function and a meromorphic function on $\mathbb{C}^2$.