Research results so far

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Until now, many fixed point approximation theorems are already known for Hilbert spaces and Banach spaces. We have conducted research to verify whether these hold true under assumptions that are considered valid in the CAT(κ) space. Hadamard space (complete CAT(0) space) is a generalization of Hilbert space in a direction different from Banach space, so this research produces a kind of generalization as a theorem. For example, in my research, [1] gives a typical example. [1] gives a finite number of maps (with reasonable assumptions) in a complete CAT(1) space, uses W-mapping construction from their convex combinations and compositions, multiplexes the iterations, and uses a common fixed We have obtained the point approximation theorem. This is an extension of [3], which obtains an approximation theorem for a common fixed point of a finite number of maps in a Banach space, and [4], which obtains an approximation theorem for a fixed point of a single map in a complete CAT(1) space. This is the result of a study conducted in the form of The assumptions for each mapping have been proven, including good concrete examples such as resolvent and nonexpansive. In addition, in [2], by applying the multiplexing of iterations in [1] in the complete CAT(1) space to a fixed point approximation method called the contraction projection method or the CQ projection method, He has already succeeded in developing a well-known theorem in Hilbet space into a theorem in CAT(1) space. Note that [2] assumes that a finite number of mappings are nonexpandable. Although this is a valid property in existing results in Hilbert space and Banach space, we have not been able to give an important example of nonexpansive mapping in CAT(1) space. Therefore, it is hoped that the theorem for mapping will be developed under assumptions that include many concrete examples (for example, strongly quasi-nonexpansive and Δ-demiclosed). Although not mentioned in the paper [2] and unpublished, the resolvent in CAT(1) defined by Kimura and Takasaka (which has the properties of strongly quasinonexpansive and Δ-demiclosed) has its inherent properties. By using also the result of [2] can be proved.

References

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