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Summary of my previous researches

I study the stability of solitary waves for nonlinear dispersive equations. Nonlinear dispersive equations appear in various fields as mathematical models describing dispersive and nonlinear wave phenomena, including nonlinear Schrödinger equations and KdV equation as representative examples. Solitary waves are solutions that propagate at a constant frequency and speed without changing their shape, and they play important role in the physical and mathematical characteristics of the equations. Therefore, it is important to investigate the properties of solitary waves, especially their stability, in order to understand the time global behavior of general solutions.

The results of my main papers are described below.

Instability of two-paragraph family of solitary waves in degenerate cases [2]. The abstract theory of Grillakis–Shatah–Strauss (1987, 1990) has been widely applied to determine the stability of solitary waves, but this theory cannot be applied to degenerate cases. In this paper, we establish an abstract theory of instability of two-parameter families of solitary waves in the degenerate case. Moreover, we applied the theory to the generalized derivative nonlinear Schrödinger equation and proved the instability of solitary waves at the boundary between stable and unstable solitary waves.

Strong instability of solitary wave [4]. For the nonlinear Schrödinger equation with inverse square potential, previous studies have obtained strong instability of solitary waves with positive energy. This assumption is a strong assumption that can be regarded as analogous to the situation without potential. In this paper, we prove the strong instability of solitary waves under a weaker assumption that is natural from the scaling point of view. The methods of this paper can be applied to various equations with similar scaling properties.

Instability of algebraic standing waves [6]. The double power nonlinear Schrödinger equation has not only exponentially decaying standing waves, but also polynomially decaying ones. Since algebraic standing waves appear in a critical cases, abstract theory cannot be applied to them. In this paper, we use energy and scaling analysis, prove the strong instability of the algebraic solitary wave when the nonlinear term is mass-critical and mass-supercritical, and give sufficient conditions for the exponent of the nonlinear term to make the algebraic solitary wave unstable in the mass-subcritical case.

2-dimensional nonlinear Schrödinger equation with point interaction [8]. The nonlinear Schrödinger equation with point interaction (delta potential) has been the subject of much research on the stability of solitary waves in one dimension. However, in two dimensions, the equation has not been studied due to difficulties such as the difference of the energy space from the usual Sobolev space. In this paper, we show local well-posedness in the two-dimensional case using the energy method and Strichartz estimates. We also showed the existence of solitary waves, its positivity, symmetry, uniqueness, and nondegeneracy. Furthermore, by using the perturbation argument, we showed the stability of solitary waves with small frequencies, and the stability/instability of solitary waves with large frequencies.