I am working on $(\mathfrak{g}, K)$**-modules over commutative rings and related geometry**. The concept of $(\mathfrak{g}, K)$-modules is an algebraic model of representations of real reductive Lie groups. Motivated by number theory and mathematical physics, studies on $(\mathfrak{g}, K)$-modules over commutative rings have been studied since the 2010's.

My first reach is on **cohomologically induced modules over commutative rings**. In the theory of $(\mathfrak{g}, K)$-modules over the field $\mathbb{C}$ of complex numbers, the cohomological induction for a morphism $(\mathfrak{q}, M) \to (\mathfrak{g}, K)$ of Harish-Chandra pairs is the right derived functor of the right adjoint functor $I^{\mathfrak{g},K}_{\mathfrak{q},M}$ of the restriction functor from the category of $(\mathfrak{g}, K)$-modules to that of $(\mathfrak{q}, M)$-modules. This derived functor supplies us important representations like principal series representations and cohomologically induced modules.

I introduced a general definition of $(\mathfrak{g}, K)$**-modules over commutative rings and established their basic theory**, based on categorical insights to the theory over $\mathbb{C}$. In particular, I constructed the right adjoint functor $I^{\mathfrak{g},K}_{\mathfrak{q},M}$ and its right derived functor (Paper 4, Preprint 5). In Papers 3 and 5, I **worked on the flat base change properties of the right derived functor**. In Paper 5, I gave affirmative results under certain conditions. In Paper 3, I gave a counterexample when the groud ring is the ring $\mathbb{Z}$ of integers. In this counterexample, I found that a certain integral model of the induction which should give principal series representations provides us an integral model of a discrete series representation. In particular, we found **phenomena over** $\mathbb{Z}$ **that do not occur over** $\mathbb{C}$.

Incidentally, some models have smaller rings of definition which are not obtained from the cohomological induction over commutative rings. **Such smaller rings are expected to be important in applications to number theory**. Together with Fabian Januszewski, I turn my eyes to **the geometric construction**: The cohomologically induced module is isomorphic to the global section module of the twisted D-module theoretic direct image of the equivariant line bundle on the "corresponding" closed K-orbit on the partial flag variety of $\mathfrak{g}$. We focused on the fact that this construction relies on geometric operations. We thought that **the smaller ring the orbit and the line bundles are defined over, the smaller ring the resulting module is defined over**. We ran this idea. Firstly, **I worked on the descent problems on the rings of definition of partial flag schemes and equivariant line bundles on them** (Paper 2). Then we introduced the notion of **stable parabolic subgroups**, and solved the problem of the ring of definition of the orbit decomposition of their moduli space. In particular, **we solved the descent problem of the rings of definition of the closed orbits corresponding to cohomologically induced modules**. **We also established the theory of twisted D-mdoules over general base schemes**. As a result, **we obtain** $\mathbb{Z}[1/2]$**-forms of cohomologically induced modules** (Preprint 4). The **I proved that they are free as** $\mathbb{Z}[1/2]$**-modules** (Preprint 3). As an application of its proof, **we proved finiteness of their** $(\mathfrak{g}, K)$**-cohomology** (Preprint 4).

In Paper 1, I worked on the descent problem of rings of definition of non-closed orbits. I found the standard $\mathbb{Z}[1/2]$-forms of all $\mathrm{SO}(3, \mathbb{C})$-orbits on the flag variety of $\mathrm{SL}_3(\mathbb{C})$. I proved that **they are imbedded affinely over** $\mathbb{Z}[1/2]$, and that **these** $\mathbb{Z}[1/2]$**-forms exhibit a set theoretic decomposition of the flag scheme of** SL_3 **over** $\mathbb{Z}[1/2]$ (**the** $\mathbb{Z}[1/2]$**-form of the** K**-orbit decomposition of the flag scheme**).

In Preprint 2, I started to work on **basic study of contractions families** from the perspectives of abstract algebraic geometry. Here a contraction family is a certain one-parameter family obtained by replacing a structure constant of Lie groups or Lie algebra. I also introduced the quotient of contraction group schemes (e.g., of contraction families of symmetric pairs), and studied basic structures of the quotient spaces. The quotint attaches **new varieties (manifolds) which connect different symmetric varieties (spaces)**. They are expected to give new insights into studies on symmetric spaces and special functions on them in the future.

In Preprint 1, I **classified irreducible representations of quasi-reductive algebraic supergroups, and determined the division superalgebras of their endomorphisms**. I examined them for fundamental examples of real quasi-reductive algebraic supergroups.