Summary of research results so far

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In analysis for random functions I gave results as follows:

(1) Ogawa integrability ([4])

[6] shows that the Ogawa integral of the Itô process is given with the Itô integral. I and Tetsuya Kazumi showed that the Ogawa integral of the S-type Itô process, which is a noncausal extension of the Itô process, is given with the Skorokhod integral and its adjoint operator.

(2) Identification of random functions from the SFCs ([1, 2, 5])

Let X_t, $t \in [0, L]$ be a random function $X_t = \int_0^t a(t)\, dB_t + \int_0^t b(t)\, dt$ driven by a Brownian motion B. To the question whether the coefficients $a(t)$ and $b(t)$ are determined by the stochastic Fourier coefficients (SFCs for short):

$$(e_n, dX) = \int_0^L e_n\, dX$$

with respect to a CONS $(e_n)_{n\in\mathbb{N}}$ of $L^2([0, L])$ posed in [7], we got the following affirmative answer, employing the SFCs in the case that $\int dB$ is the Skorokhod integral (SFC-Ss in abbr.) and the SFCs in the case that $\int dB$ is the Ogawa integral (SFC-Os in abbr.): here, by FVP, $\mathcal{L}^{r,2}$ we mean the totality of random functions of bounded variation, totality of square itegrable Wiener functionals with differentiability index r, respectively.

- Derivation of random functions from SFC-Ss ([2, 5])
  - Derivation of $a(t) \in \mathcal{L}^{1,2}$ and $b(t) \in \mathcal{L}^{0,2}$ ([5]) (extension of the results in [9, 10])
- Derivation of random functions from SFC-Os ([1, 2, 5])
  - Derivation of $a(t)$ and $b(t)$ in the case $a(t)$ is written as $a(t) = V_t + M_t + Z_t + W_t$ with $V_t \in$ FVP, an Itô integral process M_t, a Skorokhod integral process Z_t and the Hilbert-Schmidt transform W_t of a functional in $\mathcal{L}^{1,2}$, or a more general random function (extension of the result in [8, 11])

(3) Stochastic differentiability ([3])

The stochastic derivative and quadratic variation is the fundamental operations as the inverse of the stochastic integral. However the sum of random functions X and Y with the quadratic variations does not necessarily have the quadratic variation in general. Then, given a random function V with the quadratic variation, the class:

$$Q(V) = \left\{ X : \text{random function} \;\middle|\; [X],\ \frac{dX}{dV} \text{ exists and } \frac{d[X]}{d[V]} = \left|\frac{dX}{dV}\right|^2 \right\}$$

of random functions with the quadratic variation $[\,]$ and stochastic derivative $\frac{d}{dV}$ with respect to V contains the totality of Itô integral processes, Stratonovich-Fisk integral processes and Skorokhod integral processes and forms a vector space. This result gives a sufficient condition for the sum of random functions X and Y to have the quadratic variation and means that we can unifiedly calculate the stochastic derivative and quadratic variation independent of theory of stochastic integral. Here, the result is used to prove the result in [1] on the identification of random functions from the SFCs mentioned in (2).

References

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