

Future studies We expect our studies naturally develop and the following must develop as challenging ones based on our characteristic experience.

- **Generalizations of special generic maps, related examples and classifications in specific situations.** We have obtained theorems on non-existence of special generic maps on manifolds such as projective spaces (Articles 4.1–4.6). This means that special generic maps may not cover wide classes of manifolds. We have previously considered some generalizations of such maps. Images of these maps are codimension 0 immersed manifolds and the preimages of points in the interiors are closed manifolds: originally the preimages are diffeomorphic to unit spheres and those of points in the boundaries are single points (Articles 4.7, 4.8). We have obtained non-trivial examples. Finding new general properties, examples and classifications are important problems to attack. We expect that these maps can be tools in projective spaces, toric manifolds and naturally generalized manifolds in transformation group theory, for example.
- **Constructing real algebraic functions and maps.** Nash, followed by Tognoli etc. says that we can know the existence of such maps in considerable cases. It is difficult to have examples with information on precise global structures, important polynomials etc.. We also need to apply existing methods, obtained methods, and methods and theory from real algebraic geometry.

Our studies are related to various fields: singularity theory, algebraic topology, differential topology, algebraic geometry, differential geometry, combinatorics etc.. We are studying related mathematical tools, existing important theory, ideas etc. via books, articles, seminars, discussions etc. actively. We present the following as more challenging and interdisciplinary problems.

- **Our studies in real algebraic and real analytic geometry?** In "Constructing real algebraic functions and maps", to find meanings of our study in such fields is difficult and important. Now, even low dimensional real algebraic sets and manifolds are hard to construct, classify etc.. Our studies are, as we think, considering higher dimensional versions of lower dimensional cases naturally or respecting our characteristic thoughts.
- **Our singularity theory applies to differential geometry and geometric analysis?** Isoparametric hypersurfaces and functions associated to them are well-known. Symmetric spaces and important sets such as antipodal sets are closely related to singularity of Morse functions in some nice cases. Recently, Atsufumi Honda, a specialist of differential geometry of curves, (hyper)surfaces and singularity theory, says that total curvatures may be closely related to special generic maps and explicit maps. These stories tell us that our singularity theory may be important in differential geometry and geometric analysis. We will first find explicit phenomena or problems motivating us to apply our singularity theory.