

Toric face ring can be seen as a generalization of Stanley-Reisner ring. In other words, Stanley-Reisner ring is a ring which connected polynomial rings according to the structure of a simplicial complex, whereas a toric face ring is a ring which connected Ehrhart rings according to the structure of a polytopal complex. For this reason, toric face rings have been extensively studied recently. However, there has been little research into what kind of properties a toric face ring with a certain special origin has. As I wrote in "Past Research Results," I showed the following fact concerning about this. A Hibi ring is also expressed as a normal affine semigroup ring. Further, there is an expression of the canonical module of the normal affine semigroup ring by Stanley, and it is expressed the canonical module as an ideal of the ring. I call this ideal the canonical ideal. Since the canonical ideal is a height 1 divisorial ideal, we can consider its inverse element in the divisor group (although this is a fractional ideal, I will refer to it as an anticanonical ideal from now on). I showed that the fiber cones of both canonical and anticanonical ideals are toric face rings. I am planning to investigate various angles, such as what kind of toric face ring appears or does not appear as such fiber cones. Also, the relationship between the commutative algebraic properties of these toric face rings and the ordered sets that defines the Hibi rings.

On the other hand, a Hibi ring is an Ehrhart ring of an order polytope, and the similarity of combinatorial properties between order and chain polytopes are inherited to their Ehrhart rings, except minor points. Furthermore, the chain polytope of a poset is the stable set polytope of the comparability graph of the poset, and comparability graph is perfect. Therefore, it is interesting how we can generalize the algebraic theory about chain polytopes, especially those related to commutative ring theory, are generalized to the stable set polytope of perfect graphs. Also, there is a notion of t -perfect graphs, which is similar to a perfect graph but slightly different, defined in relation to linear programming and combinatorial optimization. I would also like to study the relationship between commutative algebraic properties and these graphs.