

# Plan of Research

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**1. spaces of knots in manifolds** The relation between operads and embedding spaces stated in “research results” is based on embedding calculus due to Goodwillie-Weiss. This originally relates the space of embeddings and diagrams of configuration spaces and some people are studying embedding spaces other than long knots. Especially, the space of knotted circle in a manifold  $\text{Emb}(S^1, M)$  is being studied. In the paper no.1, I constructed a spectral sequence converging to cohomology of  $\text{Emb}(S^1, M)$ . Comparing to Sinha’s spectral sequence which abuts the same target, this spectral sequence has explicit  $E_2$ -page. I plan to calculate higher differentials of this spectral sequence. This spectral sequence was obtained by replacing the diagram of configuration spaces with a diagram of fat diagonals. Čech complex is naturally associated to a fat diagonal so elements of the spectral sequence will have an expression by a graph and cochain of the sphere tangent bundle of  $M$ . Using this expression, I will calculate the higher differentials. One can also associate a similar diagram of fat diagonals for the space of long knots in  $\mathbb{R}^n$ . Salvatore showed the little 2-disks operad is not formal in characteristic 2 using combinatorial model of the operad. This result is also recovered by using the diagram for “long knots of codimension one”. I want to study (non-)formality in other characteristic by doing similar computation.

**2. homotopy type of configuration spaces** It is an old problem in homotopy theory whether the ordered configuration space of points in a manifold is a homotopy invariant or not. In non-simply connected case, a counter-example is known but in simply connected case, it is unsolved. Recently, Idrissi and Campos-Willwacher proved real homotopy invariance of the configuration space by constructing an algebraic model of the space. In their construction, Poincaré dg-algebra is essentially used. This is a dg-algebra which has Poincaré duality compatible with differential and product. This dg-algebra exists only in the real (or rational) coefficient and in other coefficient, any similar notion such as ‘Poincaré  $E_\infty$ -algebra’ does not seem to be known to work. The problem is that the usual formulation of the duality on singular chain is not compatible with the product. I plan to construct an algebraic model of a manifold in any characteristic. If I manage to construct the model, I will try to prove the rational homotopy invariance of configuration spaces without transcendental methods. I will also consider the case of other coefficient.

**3. geometric application of non-simply connected rational homotopy theory** One of the important subject of rational homotopy theory is the restriction of homotopy type of compact Kähler manifold. In simply connected case, Deligne-Griffiths-Morgan-Sullivan shows the real homotopy type of compact Kähler manifold is formal. In non-simply connected case, Simpson gave some restriction on the fundamental group and Katzarkov-Pantev-Toën gave some restrictions on schematic homotopy type, using non-abelian Hodge theory. Tensor dg-categories naturally appear in non-abelian Hodge theory and in my formulation of rational homotopy theory gives explicit relation between models and invariants so I want to apply it to the restriction problem.