Background of Research In the study of vertex operator algebras, the structure of Abelian and tensor categories on modules has been explored from the mathematical and physical aspects, and recently the relation with the representation theory of quantum groups has attracted much attention. We have studied in particular the categories of modules of triplet W-algebras and non-unitary Virasoro vertex operator superalgebras. The former is a vertex operator algebra satisfying C_2-extremality and is famous for its connection with the representation theory of quantum groups at power roots of unity[1, 2]. The latter belongs to a family of W-algebras called principal W-algebras and is associated to the vertex operator algebras of affine sl_2 through the coset construction[3, 4]. These vertex operator algebras give examples of non-rational vertex operator algebras. Here, rational means that the category of modules is semisimple, and irrational means that the category of modules is not semisimple. In general, it is a very difficult problem to determine the structure of the Abelian or tensor category of the modules of an irrational vertex operator algebra. For example, when studying the structure of tensor products among modules, it is customary in rational vertex operator algebras to simply compute the product of characters of simple modules. In the non-rational case, on the other hand, it is a very difficult problem to determine the structure of the tensor product on the modules, since the structure of the tensor product cannot be determined by the computation of the characters alone. Our goal is to understand the properties of tensor categories of modules of irrational vertex operator algebras through the examples of triplet W-algebras and non-unitary Virasoro vertex prime superalgebras.

Results of Research The following is a summary of our previous work on triplet W-algebras $\mathcal{W}_{p_+,p_-}$, $\mathcal{SW}(m)$ and non-unitary Virasoro vertex operator superalgebras.

(1) We have determined the structure of the projective covers of the simple $\mathcal{W}_{p_+,p_-}$-modules[7]. The structure of these logarithmic modules were conjectures by Gaberdiel, Runkel, and Wood [5] in the case $(p_+, p_-) = (2, 3)$.

(2) We have given a proof of the non-semisimple fusion rules conjectured by Rasmussen [6] and Gaberdiel, Runkel, Wood [5], and determined the structure of the non-semisimple fusion ring [8]. Here, a non semisimple fusion ring is a commutative ring defined on the set of suitable indecomposable modules.

(3) We have determined the structure of the Abelian and tensor categories formed by the modles of the supertriplet W-algebra $\mathcal{SW}(m)$.

(4) We apply a certain deformation method to the free-field representation of vertex operators, and give a proof of the fusion rules of the non-unitary Virasoro vertex operator superalgebra [10] conjectured by [3]. This deformation method is a new method in the theory of vertex operator algebras, and we believe that it can be applied to various vertex operator algebras.

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