Research results by Masa-Hiko Saito

1. **Study on Hodge structures and deformations of algebraic varieties (1983–1993)**

   We studied Torelli type theorems on period maps of algebraic varieties, deformation theory, and Arakelov type theorems on K3 surfaces and Abelian varieties. ([42], [39], [36], [37], [35], [33].)

2. **Modell-Weil lattice of families of Abelian (1993–1999)**

   In the joint paper [30] with Sakakibara, we proved that the rank of the Mordell-Weil lattice of Jacobi manifolds associated to a family of curves of genus $g \geq 2$ on a rational surface is less than $4g + 4$ and determine the structure of the lattice. ([32], [31], [30], [23])

3. **Superstring theory and the mirror symmetry conjecture for Calabi-Yau manifolds (1998–)** We verified the Mirror symmetry conjecture for a certain Calabi-Yau manifolds predicted by string theory in mathematical physics, the Gopakumar-Vafa conjecture, and the Shimura-Taniyama conjecture. ([27], [24], [19], [21].)

4. **Differential equations of Painlevé type, Algebro-Geometric studies onIntegrable systems (2001–)**

   After the pioneering work [22] with Umemura, in [20] and in [16], [17] with Takebe and Terashima, we defined Okamoto-Panlve pairs as a pair of rational algebraic surfaces S and their anti-canonical divisor Y satisfying certain conditions and can classify them. We also succeeded in characterizing the Painlevé equations using the theory of deformations of pairs (S, Y) and the theory of local cohomology. Later, we developed the moduli theory of stable connections on projectve curves. We also showed that the Riemann-Hilbert correspondence from the moduli space of connections to the moduli space of monodromy data is a proper, surjective birational analytic map. This allows us to show the Painlevé property of nonlinear differential equations obtained from monodromy preserving deformations of the regular singular connections. Later, we obtained the above theorem for connections over algebraic curves with unramified irregular singularities and for regular singularities of fixed spectral type. ([7], [5]). In a joint paper with Carlos Simpson and Frank Loray [8], it was shown that two Lagrangian fibrations are naturally defined on the phase space of the Painlevé type VI equations and that they are transversal. In a joint paper [6] with Loray, it was shown that in the higher dimensional case of Garnier systems, the maps obtained from apparent singularities and from moduli of connections to moduli of vector bundles defines natural Lagrangian fibration, and that they are transversal to each other.

   In a joint paper [4] with Komyo, we describe in detail how to embed the moduli space of connections on $\mathbf{P}^1$ of rank two and with five regular singularities into a Hilbert scheme of two points in the total space of blowups of a straight line bundle on $\mathbf{P}^1$ using apparent singularities and their dual coordinates. In particular, it is diffucult to describe the case where the types of vector bundles jumps on the moduli space, but we could manage them in this paper.

   The classical Painlevé equations are obtained from monodromy preserving deformations of connections on $\mathbf{P}^1$ with fixed singularity types. In the joint paper [10] with Marius van der Put, we deal with ten types of the monodormy preserving deformations on such connections on $\mathbf{P}^1$. The moduli spaces of monodromy and Stokes data were constructed by classical invariant theory to each of 10 types. All of them are defined as cubic equations in three variables and can be regarded as extensions of the Fricke-Klein equation for Panlve VI types. In the joint paper [3], we investigated the symplectic structure of the moduli space of the framed logarithmic connections on Riemann surfaces. In [2], we extended results in [8] for the moduli space of certain irregular rank 2 connections and the moduli space of related parabolic bundles. ([22], [20], [13], [7], [5], [8], [6], [4], [3], [2], [1])