Research results

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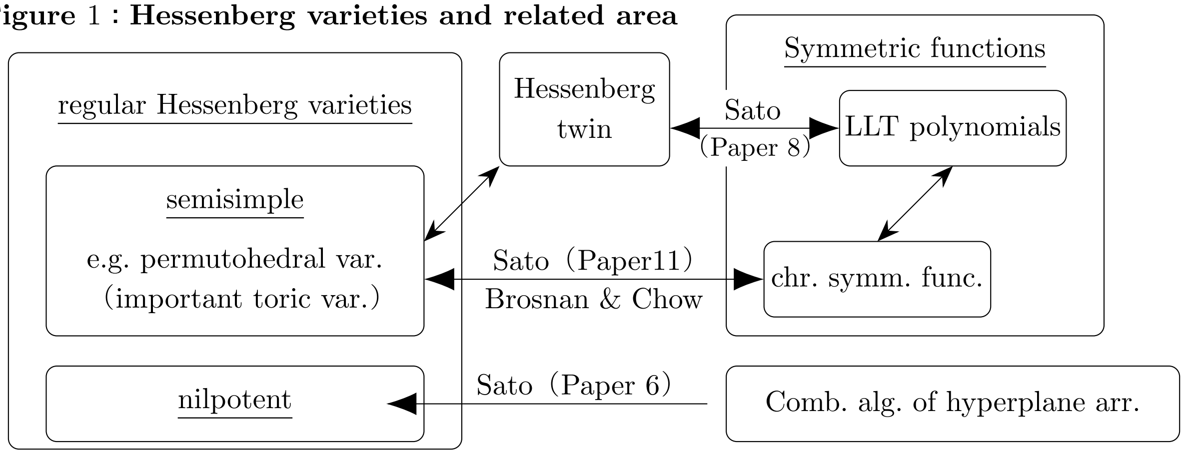
(1) Research objects

Flag varieties are important in the point of view of the symmetry of their Weyl groups and root systems. **Hessenberg varieties** are their good subvarieties which are obtained by breaking their symmetry in some sense. Hessenberg varieties give us a **geometrical way to investigate the symmetry** from a perspective outside of the symmetry. Recently I study Hessenberg varieties and some results which connect geometry, combinatorics, and representation theory. (See Figure 1)

Figure 1 : Hessenberg varieties and related area



(2) Results

First I shall explain the parallelogram in Figure 1. A Hessenberg variety is defined by some good subset of the root system and an element of the Lie algebra. The former is called a lower ideal When the Hessenberg variety is regular semisimple and of type A (we refer this condition as ($*$) later), its cohomology has an action of the symmetric group, and then it is a graded representation. There is a fact that the representation coincides with an important symmetric function, the chromatic (quasi-)symmetric function for the lower ideal (with involution). This fact (shown by Brosnan and Chow) connects geometry, combinatorics, and representation theory. Recently, I obtained an alternative, elementary proof of this fact (Paper11).

There is another class of important symmetric functions. They are (unicellular) LLT polynomials. By the way, a Hessenberg variety of ($*$) has some manifold called its twin. I found that a Hessenberg variety is to its twin what a chromatic symmetric function is to a unicellular LLT polynomial (the parallelogram in Figure 1, see Paper 8 for details). As a result, I proved that the representation of the cohomology of the twin coincides with the unicellular LLT polynomial for the lower ideal. Originally, the four objects have been studied separately. I revealed the beautiful correspondence among them.

In the case of ($*$), I gave a characterization of the lower ideal for that the cohomology ring of the Hessenberg variety is generated by degree 2 elements (Paper 7, 10), I also determined explicitly generators and relations (paper preparing).

I also determined the cohomology ring of a regular nilpotent Hessenberg variety in terms of some combinatorial algebra for the hyperplane arrangement corresponding to the lower ideal (Paper 6). This result is a generalization of the result of Borel for flag varieties.