これまでの研究成果のまとめ（英訳）

I have studied the relationship between knots and other mathematical objects in various fields. In these researches, I have used calculations of quantum invariants using knot diagrams and the skein algebra. The skein algebra is the space of quantum invariants of knots on surfaces. In particular, I have been studying the quantum $\mathfrak{g}$ invariants associated to higher simple Lie algebras $\mathfrak{g}$ other than $\mathfrak{sl}_2$. My main research can be divided into two topics. One is research on quantum invariants of knots and q-series. The other is research on the relationship between skein algebras and cluster algebras.

■Study on tails of knots and q-series The quantum $(\mathfrak{sl}_2, V_{n+1})$ invariant of the knot K, the colored Jones polynomial $J_K(n)$, is parametrized by a non-negative integer n. Here V_{n+1} denotes the $(n+1)$-dimensional irreducible representation of $\mathfrak{sl}_2$. The stability of the coefficients for this family of q-polynomials was shown independently by Armond (2013) for K is adequate, and by Garoufalidis–Lê (2015) for K is alternating. This stability of the coefficients indicates the existence of a limit $\lim_{n\to\infty} J_K(n)$ of the family of colored Jones polynomials. Such limit is given by the q-series and is called the *tail* of the knot K.

I have studied the tail of the colored $\mathfrak{g}$ Jones polynomial when $\mathfrak{g}$ is a simple Lie algebra of rank 2. In the rank 2 case, the colored $\mathfrak{g}$ Jones polynomials are parametrized by pairs of non-negative integers. I study the tail of "one-row colored" Jones polynomials; that is the limit of colored Jones polynomials colored by $(n,0)$ or $(0,n)$. In the case of $\mathfrak{g} = \mathfrak{sl}_3, \mathfrak{sp}_4$, we gave an explicit formula for the one-line colored $\mathfrak{g}$ Jones polynomial for $(2,m)$-torus knot and obtained the tail explicitly, (in [a1, a2, a4] for $\mathfrak{sl}_3$, and in [c1] for $\mathfrak{sp}_4$.) These q-series give a quantum modular form called (false) theta series. Furthermore, in the case of $\mathfrak{sl}_3$, we obtained Andrews-Gordon type identities for the (false) theta series by using two explicit formulas of the tail.

■Study on skein algebras and quantum cluster algebras for surfaces The skein algebra of a surface is defined as a quotient module of knot diagrams on the surface colored by the fundamental representation of $\mathfrak{g}$ modulo skein relations associated to $\mathfrak{g}$. It is known that the skein algebra (Kauffman bracket skein algebra) for $\mathfrak{g} = \mathfrak{sl}_2$ is related to the quantization of the ring of functions on the moduli space of flat $SL_2(\mathbb{C})$ connections on the surface. On the other hand, the cluster algebra is obtained from cluster coordinates for the moduli space of flat G-connections of surfaces. Let us denote the quantum cluster algebra for a surface Σ by $\mathscr{A}^q_{\mathfrak{g},\Sigma}$, and the skein algebra by $\mathscr{S}^q_{\mathfrak{g},\Sigma}$. Then, we conjecture the following relation: $\mathscr{S}^q_{\mathfrak{g},\Sigma}[\partial^{-1}] = \mathscr{A}^q_{\mathfrak{g},\Sigma}$. In [a5, b6] (joint work with Tsukasa Ishibashi), we proved $\mathscr{S}^q_{\mathfrak{g},\Sigma}[\partial^{-1}] \subset \mathscr{A}^q_{\mathfrak{g},\Sigma}$ $\mathfrak{g} = \mathfrak{sl}_3$ for $\mathfrak{g} = \mathfrak{sl}_3, \mathfrak{sp}_4$. In these papers, we also proved the quantum Laurent positivity for a class of skein elements called "elevation-preserving webs".

In [c2] (joint works with Tsukasa Ishibashi), we constructed isomorphisms between reduced stated skein algebras and bounded-localized skein algebras for $\mathfrak{sl}_2$, $\mathfrak{sl}_3$, and $\mathfrak{sp}_4$. In [b7] (joint works with Shunsuke Kano and Tsukasa Ishibashi), we defined the Kauffman bracket skein algebra of "walled surfaces". We showed the correspondence between the skein algebra and the cluster algebra with coefficients associated with a lamination on the surface. In [c3], we study intersection coordinates of $\mathfrak{sp}_4$-laminations via $\mathfrak{sp}_4$-skein algebras.